Pre

Please hand in your individually prepared answers on paper (not via email).

Exercises

1. Prove or disprove the following:
   (a) $\models \Diamond p \rightarrow \Box p$
   (b) $\models \Diamond (p \lor q) \rightarrow \Diamond p \lor \Diamond q$
   (c) $\models \neg \Box \neg p \rightarrow \Diamond \Diamond \neg p$
   (d) $\models \Box \Diamond p \rightarrow \Box \Box \Diamond p$

2. Consider the model defined by set of worlds $W = \{a, b, c, d\}$, the accessibility relation $R = \{(a, b), (b, c), (c, a), (d, a), (d, c)\}$, and the valuation given by $V(p) = \{a, c\}$.
   (a) Depict the model.
   (b) Give for every state if possible a modal formula characterizing that state. (You do not need to give a proof, neither for a positive nor for a negative answer.)
   (c) If possible, give a formula that is valid in the model, but not in the underlying frame. Explain your answer.

3. Let $\mathcal{F}_1 = (\{0, 1\}, \{(0, 1), (1, 0)\})$ and $\mathcal{F}_2 = (\{a, b, c, d\}, \{(a, b), (b, c), (c, d), (d, a)\})$ be frames.
   (a) Let $V_1$ be the valuation with $V_1(p) = \{0\}$ and $V_1(q) = \{1\}$.
      Give if possible a valuation $V_2$ for $\mathcal{F}_2$ such that world $0$ in $\mathcal{F}_1$ and world $a$ in $\mathcal{F}_2$ are bisimilar.
   (b) Now let $W_1$ be an arbitrary valuation on $\mathcal{F}_1$.
      Give if possible a valuation $W_2$ for $\mathcal{F}_2$ such that world $0$ in $\mathcal{F}_1$ and world $a$ in $\mathcal{F}_2$ are bisimilar.
   (c) Depict an acyclic model that is bisimilar to the model $(\mathcal{F}_1, V_1)$ with $V_1$ from (a).

4. Show that the formula $\Diamond \Box \phi \rightarrow \Box \Diamond \phi$ characterizes the frame property ‘weakly directed’ (WD) which is defined as follows:
   $\forall xyz (Rxy \land Rxz \rightarrow \exists u ( Ryu \land Rzu))$. 