advanced logic
2017 02 09
lecture 2
overview

- semantics
- game semantics
- alternative semantics
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formulas of modal logic

\[ p \mid \bot \mid \phi \rightarrow \psi \mid \Box \phi \mid \Diamond \phi \]

we could define \( \Box \) using \( \Diamond \) or vice versa:

\[ \Box \phi := \neg \Diamond \neg \phi \]
\[ \Diamond \phi := \neg \Box \neg \phi \]

we will consider different (equivalent) sets of connectives
truth and validity

$((W, R), V), w \models \phi$

$((W, R), V) \models \phi$

$(W, R) \models \phi$

$\models \phi$

what is omitted is implicitly universally quantified
local and global truth: example

if $\mathcal{M}, w \not|= \phi$ then $\mathcal{M}, w |= \neg \phi$?
local and global truth: example

if \( M, w \not\models \phi \) then \( M, w \models \neg \phi \) \( \text{yes} \)

if \( M, w \models \phi \lor \psi \) then \( M, w \models \phi \) or \( M, w \models \psi \) \( \text{no} \)
local and global truth: example

if $M, w \not\models \phi$ then $M, w \models \neg \phi$? yes

if $M, w \models \phi \lor \psi$ then $M, w \models \phi$ or $M, w \models \psi$? yes

if $M \not\models \phi$ then $M \models \neg \phi$?
local and global truth: example

if $\mathcal{M}, w \not\models \phi$ then $\mathcal{M}, w \models \neg \phi$? yes

if $\mathcal{M}, w \models \phi \lor \psi$ then $\mathcal{M}, w \models \phi$ or $\mathcal{M}, w \models \psi$? yes

if $\mathcal{M} \not\models \phi$ then $\mathcal{M} \models \neg \phi$? no

if $\mathcal{M} \models \phi \lor \psi$ then $\mathcal{M} \models \phi$ or $\mathcal{M} \models \psi$?
local and global truth: example

if $M, w \not\models \phi$ then $M, w \models \neg \phi$? yes

if $M, w \models \phi \lor \psi$ then $M, w \models \phi$ or $M, w \models \psi$? yes

if $M \not\models \phi$ then $M \models \neg \phi$? no

if $M \models \phi \lor \psi$ then $M \models \phi$ or $M \models \psi$? no
local and global truth: example

\[ W = \{ u, v, w, s \} \]
\[ R = \{(u, v), (v, w), (w, u), (s, s)\} \]
\[ V(p) = \{ w, s \} \]
\[ V(q) = \{ u, v, w \} \]

for which worlds \( x \) do we have \( M, x \models p \rightarrow \Box p \)?

for which worlds \( x \) do we have \( M, x \models \Box p \rightarrow \Diamond q \) ?

if possible give another valuation such that \( \Diamond p \rightarrow p \) is not globally true

if possible give another valuation such that \( \Diamond p \rightarrow \Box p \) is not globally true
local truth

from world to formula

definition of local truth: $\mathcal{M}, w \models \phi$

from formula to world

characterize a world via a modal formula without variables
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game semantics: setting

another approach to local truth

a model \( \mathcal{M} = ((W, R), V) \), and a world \( w \in W \), and a formula \( \phi \)

there are two players:

Verifier (V) claims that \( \phi \) is true in \( w \)

Falsifier (F) claims that \( \phi \) is false in \( w \)

a position is a pair \((w, \phi)\) with \( w \in W \) a world and \( \phi \) a formula

a move from a position \((w, \phi)\) is determined by the main operator of \( \phi \)
game semantics: position determines move

\[(t, \phi_1 \lor \phi_2)\]: \(V\) chooses a disjunct \(\phi_i\); play continues with \((t, \phi_i)\)

\[(t, \phi_1 \land \phi_2)\]: \(F\) chooses a conjunct \(\phi_i\); play continues with \((t, \phi_i)\)

\[(t, \Diamond \phi)\]: \(V\) chooses a successor \(u\) of \(t\); play continues with \((u, \phi)\)

\[(t, \Box \phi)\]: \(F\) chooses a successor \(u\) of \(t\); play continues with \((u, \phi)\)

\[(t, \neg \phi)\]: players switch roles; play continues with \((t, \phi)\)

\[(t, p)\]: if \(p\) is true in \(t\) then \(V\) wins; otherwise \(F\) wins

who should but cannot choose a successor loses
game semantics: strategy

A strategy for player $P$ is a method to select moves.

Player $P$ has a winning strategy if every path through the tree is won by $P$.

Theorem explaining the connection with the truth definition:

$\phi$ is true in $\mathcal{M}$ in $s$ iff $V$ has a winning strategy for $\mathcal{M}, s, \phi$.
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alternative definition of truth