### Overview

- **Semantics**
- **Game semantics**
- **Alternative semantics**

### Formulas of modal logic

| p  | ⊥  | φ → ψ | □φ | ◊φ |

We could define □ using ◊ or vice versa:

\[
□φ := \neg ◊ \neg φ
\]

\[
◊φ := \neg □ \neg φ
\]

We will consider different (equivalent) sets of connectives.
truth and validity

$((W, R), V), w \models \phi$

$((W, R), V) \models \phi$

$(W, R) \models \phi$

$\models \phi$

what is omitted is implicitly universally quantified

local and global truth: example

if $\mathcal{M}, w \not\models \phi$ then $\mathcal{M}, w \models \neg \phi$? yes

if $\mathcal{M}, w \models \phi \lor \psi$ then $\mathcal{M}, w \models \phi$ or $\mathcal{M}, w \models \psi$? yes

if $\mathcal{M} \not\models \phi$ then $\mathcal{M} \models \neg \phi$? no

if $\mathcal{M} \models \phi \lor \psi$ then $\mathcal{M} \models \phi$ or $\mathcal{M} \models \psi$? no

local and global truth: example

$W = \{u, v, w, s\}$

$R = \{(u, v), (v, w), (w, u), (s, s)\}$

$V(p) = \{w, s\}$

$V(q) = \{u, v, w\}$

for which worlds $x$ do we have $\mathcal{M}, x \models p \rightarrow \square p$?

for which worlds $x$ do we have $\mathcal{M}, x \models \square p \rightarrow \diamond q$?

if possible give another valuation such that $\diamond p \rightarrow p$ is not globally true

if possible give another valuation such that $\diamond p \rightarrow \square p$ is not globally true

local truth

from world to formula

definition of local truth: $\mathcal{M}, w \models \phi$

from formula to world

characterize a world via a modal formula without variables
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**game semantics**: setting

another approach to local truth

a model $\mathcal{M} = ((W, R), V)$, and a world $w \in W$, and a formula $\phi$

there are two players:

Verifier (V) claims that $\phi$ is true in $w$

Falsifier (F) claims that $\phi$ is false in $w$

a position is a pair $(w, \phi)$ with $w \in W$ a world and $\phi$ a formula

a move from a position $(w, \phi)$ is determined by the main operator of $\phi$

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**game semantics**: position determines move

$(t, \phi_1 \lor \phi_2)$: $V$ chooses a disjunct $\phi_i$; play continues with $(t, \phi_i)$

$(t, \phi_1 \land \phi_2)$: $F$ chooses a conjunct $\phi_i$; play continues with $(t, \phi_i)$

$(t, \lozenge \phi)$: $V$ chooses a successor $u$ of $t$; play continues with $(u, \phi)$

$(t, \square \phi)$: $F$ chooses a successor $u$ of $t$; play continues with $(u, \phi)$

$(t, \neg \phi)$: players switch roles; play continues with $(t, \phi)$

$(t, p)$: if $p$ is true in $t$ then $V$ wins; otherwise $F$ wins

who should but cannot choose a successor loses

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**game semantics**: strategy

a strategy for player $P$ is a method to select moves

player $P$ has a winning strategy if every path through the tree is won by $P$

Theorem explaining the connection with the truth definition:

$\phi$ is true in $\mathcal{M}$ in $s$ iff $V$ has a winning strategy for $\mathcal{M}, s, \phi$
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