overview

- more about bisimulations
- decidability
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proposition: bisimulation is an equivalence

reflexivity:
\{(w, w) \mid w \in W\} is a bisimulation

symmetry:
if $Z$ is a bisimulation, then
$Z^{-1} = \{(w, w') \mid (w', w) \in Z\}$ is a bisimulation

transitivity:
if $Z$ and $Z'$ are bisimulations, then
$Z \circ Z' = \{(u, w) \mid \exists((u, v) \in Z \land (v, w) \in Z')\}$ is a bisimulation
remark: bisimulation versus two simulations

a simulation: local harmony and only zig, no zag

question:
is $\mathcal{M}, w$ bisimilar with $\mathcal{N}, \nu$
the same as
$\mathcal{M}, w$ similar with $\mathcal{N}, \nu$ and also $\mathcal{N}, \nu$ similar with $\mathcal{M}, w$?

answer:
no!
disjoint union of models

assume models $\mathcal{M}_i = (W_i, R_i, V_i)$

we define the disjoint sum $\bigcup_i \mathcal{M}_i$ with

$W = \bigcup W_i$

$R = \bigcup R_i$

$V(p) = \bigcup V_i(p)$

then we have:
for every $i$, for every $w \in W_i$: $\mathcal{M}_i, w$ bisimilar with $\bigcup_i \mathcal{M}_i, w$
use of disjoint union

define the operator $A$ for global box as follows:

$\mathcal{M}, w \models A\phi$ if and only if $\mathcal{M}, u \models \phi$ for all $u$

claim: we cannot define $A$ in basic modal logic

that is: there is no formula $\zeta(p)$ depending on $p$ such that

$\mathcal{M}, w \models A\phi$ if and only if $\mathcal{M}, w \models \zeta(\phi)$ for all $\mathcal{M}, w$

suppose $\zeta$ exists

we work towards a contradiction using disjoint union
the bisimulation contraction of a model $W, R, V$ is the model $W', R', V'$:

$W'$ consists of equivalence classes $|s| = \{t \text{ such that } s \text{ and } t \text{ bisimilar}\}$

$R' \ |s| \ |t|$ if $Ruv$ for some $u \in |s|$ and some $v \in |t|$

$V'(p) = \{|s| \ |s \in V(p)\}$
unravelling of world \( s \) in model \( W, R, V \) is the model \( W', R', V' \):

\( W' \) consists of \((s_1, \ldots, s_n)\) with \( s_1 = s \) and \( R s_i s_{i+1} \)

\( R' \) related \((s_1, \ldots, s_n)\) to \((s_1, \ldots, s_n, s_{n+1})\) if \( R s_n s_{n+1} \)

\( V'(p) = \{ (s_1, \ldots, s_n) | s_n \in V(p) \} \)
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decidability

prop1 is decidable with an exponential algorithm

pred1 is undecidable for example via Post Correspondence Problem

modal logic is decidable
decidability of basic modal logic

finite model property

inductive analysis: sequents

semantic tableaux

translation
finite model property

suppose $\phi$ is satisfiable

then $\phi$ is satisfiable in a model with at most $f(\phi)$ worlds

so we have a decision procedure for satisfiability of $\phi$:

compute $f(\phi)$

consider all (finitely many) models with at most $f(\phi)$ worlds, up to isomorphism

see whether in one of them $\phi$ holds

$f(\phi)$ is $2^{s(\phi)}$ with $s(\phi)$ the number of subformulas of $\phi$
sequent

for propositional logic:

\( \phi_1, \ldots, \phi_n \Rightarrow \psi_1, \ldots, \psi_m \)

valid if in every model

the conjunction of the \( \phi_i \) implies the disjunction of the \( \psi_i \)

for modal logic:

\( \phi_1, \ldots, \phi_n \Rightarrow \psi_1, \ldots, \psi_m \)

valid if in every model, in every world in that model

the conjunction of the \( \phi_i \) implies the disjunction of the \( \psi_i \)
reducing modal sequents: propositional part

$p, q_1, \ldots, q_n \Rightarrow p, r_1, \ldots, r_m$ is valid

$A, \neg \phi \Rightarrow B$ if and only if $A \Rightarrow \phi, B$

$A \Rightarrow \neg \phi, B$ if and only if $A, \phi \Rightarrow B$

$A, \phi \land \psi \Rightarrow B$ if and only if $A, \phi, \psi \Rightarrow B$

$A \Rightarrow \phi \land \psi, B$ if and only if both $A \Rightarrow \phi, B$ and $A \Rightarrow \psi, B$

$A, \phi \lor \psi \Rightarrow B$ if and only if $A, \phi \Rightarrow B$ and $A, \psi \Rightarrow B$

$A \Rightarrow \phi \lor \psi, B$ if and only if $A \Rightarrow \phi, \psi, B$

gives a decision procedure for propositional logic
extension to modal logic

we get a sequent of the form
\[ p_1, \ldots, p_n, \Diamond \phi_1, \ldots, \Diamond \phi_m \Rightarrow q_1, \ldots, q_k, \Diamond \psi_1, \ldots, \Diamond \psi_k \]

such a sequent is valid if and only if

either \( p_i = q_j \) for some \( i \) and \( j \)

or \( \phi_i \Rightarrow \psi_1, \ldots, \psi_k \) is valid for some \( i \)
decidability using sequents

rewrite formula

rewrite sequent

decide on validity of sequent
example

$\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$