overview

more about bisimulations

decidablity

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proposition: bisimulation is an equivalence

reflexivity:
\{(w, w) \mid w \in W\} is a bisimulation

symmetry:
if \(Z\) is a bisimulation, then
\(Z^{-1} = \{(w', w) \mid (w, w') \in Z\}\) is a bisimulation

transitivety:
if \(Z\) and \(Z'\) are bisimulations, then
\(Z \circ Z' = \{(u, w) \mid \exists (u, v) \in Z \land (v, w) \in Z'\}\) is a bisimulation
remark: bisimulation versus two simulations

a simulation: local harmony and only zig, no zag

question:
is $\mathcal{M}, w$ bisimilar with $\mathcal{N}, v$
the same as
$\mathcal{M}, w$ similar with $\mathcal{N}, v$ and also $\mathcal{N}, v$ similar with $\mathcal{M}, w$?

answer:
no!

use of disjoint union

define the operator $A$ for global box as follows:
$\mathcal{M}, w \models A\phi$ if and only if $\mathcal{M}, u \models \phi$ for all $u$

claim: we cannot define $A$ in basic modal logic
that is: there is no formula $\zeta(p)$ depending on $p$ such that
$\mathcal{M}, w \models A\phi$ if and only if $\mathcal{M}, w \models \zeta(\phi)$ for all $\mathcal{M}, w$
suppose $\zeta$ exists
we work towards a contradiction using disjoint union

disjoint union of models

assume models $\mathcal{M}_i = (W_i, R_i, V_i)$
we define the disjoint sum $\bigcup_i \mathcal{M}_i$ with
$W = \bigcup W_i$
$R = \bigcup R_i$
$V(p) = \bigcup V_i(p)$

then we have:
for every $i$, for every $w \in W_i$:
$\mathcal{M}_i, w$ bisimilar with $\bigcup_i \mathcal{M}_i, w$

bisimulation contraction

the bisimulation contraction of a model $W, R, V$ is the model $W', R', V'$:
$W'$ consists of equivalence classes $|s| = \{ t \text{ such that } s \text{ and } t \text{ bisimilar} \}$
$R' |s||t| \text{ if } Ruv \text{ for some } u \in |s| \text{ and some } v \in |t|$
$V'(p) = \{ |s| \mid s \in V(p) \}$
unravelling of world $s$ in model $W, R, V$ is the model $W', R', V'$:

$W'$ consists of $(s_1, \ldots, s_n)$ with $s_1 = s$ and $Rs_i; s_{i+1}$

$R'$ related $(s_1, \ldots, s_n)$ to $(s_1, \ldots, s_n, s_{n+1})$ if $Rs_n; s_{n+1}$

$V'(p) = \{(s_1, \ldots, s_n) | s_n \in V(p)\}$

decidability

prop1 is decidable with an exponential algorithm

pred1 is undecidable for example via Post Correspondence Problem

modal logic is decidable

decidability of basic modal logic

finite model property

inductive analysis: sequents

semantic tableaux

translation
finite model property

suppose $\phi$ is satisfiable
then $\phi$ is satisfiable in a model with at most $f(\phi)$ worlds
so we have a decision procedure for satisfiability of $\phi$:
compute $f(\phi)$
consider all (finitely many) models with at most $f(\phi)$ worlds, up to isomorphism
see whether in one of them $\phi$ holds
$f(\phi)$ is $2^{s(\phi)}$ with $s(\phi)$ the number of subformulas of $\phi$

reducing modal sequents: propositional part

$p, q_1, \ldots, q_n \Rightarrow p, r_1, \ldots, r_m$ is valid
$A, \neg \phi \Rightarrow B$ if and only if $A \Rightarrow \phi, B$
$A \Rightarrow \neg \phi, B$ if and only if $A, \phi \Rightarrow B$
$A, \phi \land \psi \Rightarrow B$ if and only if $A, \phi, \psi \Rightarrow B$
$A \Rightarrow \phi \land \psi, B$ if and only if both $A \Rightarrow \phi, B$ and $A \Rightarrow \psi, B$
$A, \phi \lor \psi \Rightarrow B$ if and only if $A, \phi \Rightarrow B$ and $A, \psi \Rightarrow B$
$A \Rightarrow \phi \lor \psi, B$ if and only if $A \Rightarrow \phi, \psi, B$
gives a decision procedure for propositional logic

extension to modal logic

we get a sequent of the form
$p_1, \ldots, p_n, \Diamond \phi_1, \ldots, \Diamond \phi_m \Rightarrow q_1, \ldots, q_k, \Diamond \psi_1, \ldots, \Diamond \psi_k$
such a sequent is valid if and only if
either $p_i = q_j$ for some $i$ and $j$
or $\phi_i \Rightarrow \psi_1, \ldots, \psi_k$ is valid for some $i$

sequent

for propositional logic:
$\phi_1, \ldots, \phi_n \Rightarrow \psi_1, \ldots, \psi_m$
valid if in every model
the conjunction of the $\phi_i$ implies the disjunction of the $\psi_i$

for modal logic:
$\phi_1, \ldots, \phi_n \Rightarrow \psi_1, \ldots, \psi_m$
valid if in every model, in every world in that model
the conjunction of the $\phi_i$ implies the disjunction of the $\psi_i$
decidability using sequents

rewrite formula

rewrite sequent

decide on validity of sequent

\[ \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q) \]