overview

- temporal frames
- basic temporal logic
practical remarks

no midterm exam

please note the room of the exercise class on Feb 28

for those who are interested: hash code google
overview

- temporal frames
- basic temporal logic
the book so far

basic modal logic
slides and MLOM chapter 2

bisimulations
slides and MLOM chapter 3

decidability
slides and MLOM chapter 4

extensions global box, difference, until
slides and MLOM chapter 7.4

temporal logic
mainly the slides, also chapter by Yde Venema
different approaches to temporal logic

linear time logic
Pnueli

branching time logic
Ben-Ari, Manna, Pnueli, Clarke and Emerson

more temporal logic
Kupferman, Venema
temporal logic: linear time and branching time

Moshe Vardi
paper

Rob van Glabbeek
paper
temporal model: definition

first approach to temporal logic: a special case of basic modal logic

a frame \( F = (T, <) \) is a \textbf{temporal frame} if

\(< \) is irreflexive: not \( t < t \) for all \( t \), and

\(< \) is transitive: if \( t < u \) and \( u < v \) then \( t < v \)

so \( (\mathbb{N}, <) \) is a temporal frame

a \textbf{temporal model} is

a temporal frame \( (T, <) \) with a valuation \( V : \text{Var} \rightarrow \mathcal{P}(T) \)
remark irreflexivity

recall: irreflexivity is not modally definable (lecture 4)

we can show this also using

$\mathcal{A} = (\{a\}, \{(a, a)\})$ and

$\mathcal{N} = (\mathbb{N}, \{(n, n + 1) \mid n \in \mathbb{N}\}$
temporal model: example

\[ M = (\mathbb{N}, <, V) \text{ with } V(q) = \{ n \mid n \geq 1000 \} \text{ and } V(r) = \{ 2n \mid n \in \mathbb{N} \} \]

then we have:

\[ 0 \vDash \Diamond \Box q \]

\[ n \not\vDash \Diamond \Box r \quad \text{for an arbitrary } n \in \mathbb{N} \]

\[ M \vDash \Box \Diamond r \]

\[ M \vDash \Box \Diamond \neg r \]
properties of temporal frames

we consider some properties of temporal frames

with the intuition of ‘time’ in mind

some are definable in basic modal logic and some are not
right-linearity

intuition: all future points are related

definition: \((x < y) \land (x < z) \rightarrow (y < z) \lor (y = z) \lor (z < y)\)

right-linearity is modally definable (see exercise class 4)

by \((\Diamond p \land \Diamond q) \rightarrow (p \land \Diamond q) \lor (p \land q) \lor (\Diamond p \land q)\)

and also by \(\Box((p \land \Box p) \rightarrow q) \lor \Box((q \land \Box q) \rightarrow p)\)
right-branching

intuition: right-branching is not-right-linear
so some point has two unrelated points in the future

definition: there exist $x, y, z$ such that $x < y$ and $x < z$ but
$\neg(y < z) \land y \neq z \land \neg(z < y)$

right-branching is not modally definable

why?
intuition: every point with a successor has an immediate successor

definition: \((x < y) \rightarrow \exists z : x < z \land \neg \exists u : (x < u) \land (u < z)\)

discreteness is modally definable in basic temporal logic (later)
intuition: between any two points is a third one

definition: \( x < z \rightarrow \exists y \ (x < y \land y < z) \)

density is modally definable: by \( \Diamond p \rightarrow \Diamond \Diamond p \)

neither dense nor discrete: \( \{0, 1\} \cup [2, 3] \) with normal \( < \) ordering

both dense and discrete: \( \{0\} \) with empty relation
example

temporal frame: \((\{0, 1\} \cup [2, 3], <)\) with < as usual

not dense: there is no \(x\) such that \(0 < x < 1\)

not discrete: 2 has no immediate successor

temporal frame \((\{0\} \cup \{2^{-n} \mid n \in \text{nat}\}, <)\) with < as usual

not dense: there is no \(x\) such that \(2^{-1} < x < 1\)

not discrete: 0 has a successor (for example 1) but no immediate successor

temporal frame \((\{0\}, \emptyset)\) is both dense and discrete
new operators

we will consider new(?) operators with a time-intuition

next:  \( \bigotimes \)

\[ \mathcal{M}, t \models \bigotimes \phi \text{ iff } \exists v \ t < v \land (\neg \exists u : t < u < v) \land \mathcal{M}, v \models \phi \]

next is not definable in basic modal logic

until:  \( U \)

\[ \mathcal{M}, t \models \phi U \psi \text{ iff } \exists v : t < v \land \mathcal{M}, v \models \psi \land \forall u : t < u < v \rightarrow \mathcal{M}, u \models \phi \]

until is not definable in basic modal logic
diamond and next and until

in a temporal frame

◊p is equivalent to ⊤Up

⊗p is equivalent to ⊥Up
until is not definable in basic modal logic: idea

suppose that $U$ is definable in basic modal logic

then there is a BML formula $\zeta(a, b)$ such that

$\mathcal{M}, t \models \phi U \psi$ if and only if $\mathcal{M}, t \models \zeta(\phi, psi)$

for all pointed models, for all formulas

now try to construct bisimilar $\mathcal{M}, a$ and $\mathcal{M}', a'$ such that

$\mathcal{M}, a \models pUq$ and $\mathcal{M}', a' \not\models pUq$
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basic temporal logic

second approach to temporal logic

formulas inductively defined by:

\[ p \mid \perp \mid \neg \phi \mid \phi \land \psi \mid \langle F \rangle \phi \mid \langle P \rangle \phi \]

truth and validity for basic temporal formulas in temporal frames

\[ \mathcal{M}, t \models p \text{ iff } t \in V(p) \]

\[ \mathcal{M}, t \models \perp \text{ never} \]

\[ \mathcal{M}, t \models \neg \phi \text{ iff } \mathcal{M}, t \not\models \phi \]

\[ \mathcal{M}, t \models \phi \land \psi \text{ iff } \mathcal{M}, t \models \phi \text{ and } \mathcal{M}, t \models \psi \]

\[ \mathcal{M}, t \models \langle F \rangle \phi \text{ iff for some } u \text{ with } t < u \text{ we have } \mathcal{M}, u \models \phi \]

\[ \mathcal{M}, t \models \langle P \rangle \phi \text{ iff for some } s \text{ with } s < t \text{ we have } \mathcal{M}, s \models \phi \]
basic temporal logic: remark

we can define $[F]\phi$ as $\neg \langle F \rangle \neg \phi$ and $[P]\phi$ as $\neg \langle P \rangle \neg \phi$

alternatively we can define

$\mathcal{M}, t \models [F]\phi$ iff for all $u$ with $t < u$ we have $\mathcal{M}, u \models \phi$

$\mathcal{M}, t \models [P]\phi$ iff for all $s$ with $s < t$ we have $\mathcal{M}, s \models \phi$