1. Show the following in some detail:
   (a) $5n^2 + 3n \log n + 2n + 5 \in O(n^2)$
   (b) $20n^3 + 10n \log n + 5 \in O(n^3)$
   (c) $3 \log n + 2 \in O(\log n)$
   (d) $2^{n^2} \in O(2^n)$,
   (e) $2n + 100 \log n \in O(n)$.

2. Give a concrete example of $f_1$ and $f_2$ such that $f_1(n) \in O(g_1(n))$ and
   $f_2(n) \in O(g_2(n))$ but $f_1(n) - f_2(n) \notin O(g_1(n) - g_2(n))$.

3. Describe a recursive algorithm for finding the maximum element in an
   array of $n$ elements. Analyse the worst-case time complexity of your al-
   gorithm.

4. Do we have $2^{n+1} \in O(2^n)$? Do we have $2^{2n} \in O(2^n)$?

5. Show the following: if $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$ then $f_1(n) +
   f_2(n) \in O(g_1(n) + g_2(n))$.

6. Depict the max-heap $[16, 14, 10, 8, 7, 9, 3, 2, 4, 1]$ as a tree.

7. Give the array representation of the max heap of the following picture:

8. Is the array
   
   $[23, 17, 14, 6, 13, 10, 1, 5, 7, 12]$ 
   
   a max-heap?

9. Is an array of decreasing numbers always a max-heap?
10. Show that in any subtree of a max-heap, the root of the subtree contains the largest value occurring anywhere in that subtree.

11. Where in a max-heap is the smallest element, assuming that all elements are distinct?

12. What are the minimum and maximum numbers of elements in a heap of height \( h \)?

13. Show that an \( n \)-element heap has height \( \lceil \log(n) \rceil \).

14. Turn the following sequences into a heap, using the procedure `downMaxHeap` from the slides which is the same as the procedure `MaxHeapify` from the book. First determine (by hand) the index of the node that should be bubbled. Use the Figure in the book as model.

   1  8  6  5  3  7  4
   27 17  3 16 13 10  1  5  7 12  4  8  9  0

15. Provide pseudo-code for a procedure `downMinHeap` or in book-terminology `MinHeapify`, similar to `downMaxHeap` or `MaxHeapify` that takes as input an array \( A \) and an index \( i \) in \( A \), and that let the key at \( i \) bubble down to restore the min-heap property.

16. Illustrate the operation of `buildMaxHeap` on the sequence

   5  3 17 10 84 19 6 22 9

   using Figure 6.3 from the book as model.

17. Use heapsort to sort the following sequences; use Figure 6.4 in the book as model.

   1  2  3  4  5
   5  4  3  2  1
   5 13  2 25  7 17 20  8  4

18. Describe an algorithm for checking whether an array of numbers is a max-heap, and determine its worst-case time complexity.

19. Give a definition of a ternary max-heap.