1. Illustrate the operation that removes and returns the maximum element on the max-heap $A = [15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1]$.

2. Let $A = [15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1]$ be a max-heap. Illustrate the operation of adding 10.

3. Apply quicksort to the following input:

   8 5 2 7 1 3 4 6

   Illustrate at least two runs of partition.

4. Partition can be implemented in many different ways.
   Consider for example a variation where we use $A[p]$ as pivot instead of $A[r]$.
   You may consider even two different solutions: one where you only manipulate with the pivot initially, and one where you consider a ‘mirrored version’ of partition.

5. Here is an example of an alternative implementation of partition.

   **Algorithm** partition($A, l, r$):
   
   $i := l - 1$
   $j := r + 1$
   
   while $i < j$ do
   
   $i := i + 1$
   
   while $A[i] \leq A[l]$ and $i \leq r$ do
   
   $i := i + 1$
   
   $j := j - 1$
   
   while $A[j] > A[l]$ and $j > l$ do
   
   $j := j - 1$
   
   if $i < j$ then
   
   swap($A[i], A[j]$)
   swap($A[l], A[i - 1]$)
   
   return $i - 1$
Apply this algorithm to the input we already used above:

\[ 8 \ 5 \ 2 \ 7 \ 1 \ 3 \ 4 \ 6 \]

or, maybe better, to

\[ 3 \ 6 \ 1 \ 7 \ 8 \ 2 \ 5 \ 4 \ 6 \]

6. We use quicksort and partition as given in the book.
   Give an example of a best-case input of length 7 for quicksort. Also draw the recursion tree.

7. We use quicksort and partition as given in the book.
   Give an example of a worst-case input for quicksort. Also draw the recursion tree.

8. Adapt quicksort (as given in the book) so that it sorts sequences in non-increasing (instead of non-decreasing) order.

9. Explain why the running time of partition (as given in the book) is in \( \mathcal{O}(n) \).

   (Remark: even in \( \Theta(n) \).)

10. Determine for each of the following functions \( f \) and \( g \) whether \( f \in \mathcal{O}(g) \) and/or \( g \in \mathcal{O}(f) \).

   (a) \( f(n) = 5n^2 + 3n + 7 \) and \( g(n) = n^3 \).
   (b) \( f(n) = \sum_{i=1}^{n} i \) and \( g(n) = n^2 \).
   (c) \( f(n) = n^n \) and \( g(n) = n! \).
   (d) \( f(n) = n \log_2 n \) and \( g(n) = n\sqrt{n} \).