



1. (a) Show that $(B \rightarrow (A \rightarrow B) \rightarrow C) \rightarrow B \rightarrow C$ is a tautology of minimal prop1.

(b) Give the type derivation in simply typed λ -calculus corresponding to the proof of 1.(a).

2. (a) Show that $(A \rightarrow A \rightarrow B) \rightarrow A \rightarrow B$ is a tautology of minimal prop1.

(b) Give the type derivation in simply typed λ -calculus corresponding to the proof of 2.(a).

3. (a) Show that the formula $((A \rightarrow B \rightarrow A) \rightarrow B) \rightarrow B$ is a tautology of minimal prop1.

(b) Give the type derivation in simply typed λ -calculus corresponding to the proof of 3.(a).

4. (a) Show that $((A \rightarrow B) \rightarrow C \rightarrow D) \rightarrow C \rightarrow B \rightarrow D$ is a tautology of minimal prop1.

(b) Give the type derivation in simply typed λ -calculus corresponding to the proof of 4.(a).

5. (a) What is the definition of a detour in a natural deduction proof?

(b) Give a proof of $A \rightarrow A \rightarrow A$ in first-order minimal propositional logic that contains a detour.

(c) Give the λ -term that corresponds to the proof of 5.(b).
Which part corresponds to the detour?
Give the normal form of the λ -term.

6. Show that the following formulas are tautologies of prop1:

- (a) $((A \rightarrow \perp) \rightarrow A) \rightarrow A \rightarrow \neg\neg A \rightarrow A$,
- (b) $\neg\neg((A \rightarrow B) \rightarrow A) \rightarrow A$,
- (c) $(A \vee \neg A) \rightarrow ((\neg A \rightarrow B) \wedge (\neg A \rightarrow \neg B)) \rightarrow A$,
- (d) $\neg\neg((A \vee \neg A) \rightarrow ((\neg A \rightarrow B) \wedge (\neg A \rightarrow \neg B)) \rightarrow A)$.

7. Replace in the following terms the '?'s by simple types, such that we obtain typable λ -terms.

- (a) $\lambda x:?. \lambda y:?. x$
- (b) $\lambda x :?. \lambda y :?.(x y)$
- (c) $\lambda x:?. \lambda y:?. x y y$
- (d) $\lambda x:?. \lambda y:?. x (x y)$
- (e) $\lambda x:?. \lambda y:?. \lambda z:?. x (y z)$
- (f) $\lambda x:?. \lambda y:?. \lambda z:?. y (\lambda u:?. x)$
- (g) $\lambda x :?. \lambda y :?. \lambda z :?. (\lambda u :?. y) x z$
- (h) $\lambda x :?. \lambda y :?. x ((\lambda z :?. y)y)$
- (i) $\lambda x:?. \lambda y:?. \lambda z:?. z ((\lambda u:?. y) x)$

8. Give four different closed normal forms of type $(A \rightarrow A) \rightarrow A \rightarrow A$.