C3.7 Multi-parent Recombination

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Abstract

In this section we survey recombination operators that can utilize more than two parents to create offspring. Some multi-parent recombination operators are defined for a fixed number of parents, e.g. have arity three, while in some operators the number of parents is a random number that might be greater than two, and in yet other operators the arity is a parameter that can be set to an arbitrary integer. We pay special attention to this latter type of operators and summarize results on the effect of operator arity on evolutionary algorithm performance.

C3.7.1 Introduction

To make the coming survey unambiguous we have to start with setting some conventions on terminology. The term *population* will be used for a multiset of individuals that undergoes selection and reproduction. This terminology is maintained in genetic algorithms, evolutionary programming and genetic programming, but in evolution strategies all μ individuals in a (μ, λ) or $(\mu + \lambda)$ strategy are called parents. We, however, use the term *parents* only for those individuals that are selected to undergo recombination. In other words, parents are those individuals that are actually used as inputs for a recombination operator; the *arity of a recombination operator* is the number of parents it uses. The next notion is that of a *donor*, being a parent that actually contributes to (at least one of) the alleles of the child(ren) created by the recombination operator. This contribution can be for instance the delivery of an allele, as in uniform crossover in canonical GAs, or the participation in an averaging operation, as in intermediate recombination in ES. As an illustration consider a steady-state GA where 100 individuals form the population and two of them are chosen as parents to undergo uniform crossover to create one single offspring. If, by pure chance, the offspring only inherits alleles from parent 1, then parent 1 is a donor, and parent 2 is not.

C3.7.2 Miscellaneous operators

We begin this survey with papers where the multi-parent aspect has an incidental character. By an incidental character we mean that the operator is defined and used in a specific application and has, for instance, a certain fixed arity, or even if the definition is general and would allow comparison between different number of parents, this aspect is not given attention.

The recombination mechanism of Kaufman (1967) is applied for evolving models for a given process, where a model is an array of a number of blocks, and models may differ in the numbers of blocks they contain. Recombination of four models to create one new model is defined as follows. The size of the child (the number of blocks) equals the size of each of its parents with probability 0.25. The i-th block of the child is chosen with equal probability from those parents that have at least i blocks. Let us note that there is an exception of this latter rule of choosing one of the parents blocks, but that exception has a very problem-specific reason, therefore we rather present the general idea here.

In an extensive study on bit vector function optimization stochastic iterated genetic hill-climbing (SIGH) is studied and compared with other techniques, such as GAs, iterated hill-climbing and simulated annealing (Ackley 1987). SIGH applies a sophisticated probabilistic voting mechanism with time-dependent probability distributions (cooling), where m "voters" (m being the size of the population) determine the values of a new bit-string. SIGH is shown to be better than a GA with 1-point and uniform crossover on four out of the six test functions and the overall conclusion is that it is "competitive in speed with a variety of existing algorithms".

In the introductory paper on the parallel genetic algorithm ASPARAGOS (Mühlenbein 1989), *p-sexual voting recombination* is applied for the quadratic assignment problem. Let us remark that the name p-sexual is somewhat misleading, as there are no different genders and no restriction on having one representative of each gender for recombination. The voting recombination produces one child of p parents based on a threshold value v. It determines the i-th allele of the child by comparing the i-th alleles of the selected parent individuals. If the same allele is found more often than the threshold v, this allele is included in the child, other bits are filled in randomly. In the experiments the values $p = 7$ and $v = 5$ are used and it "worked surprisingly well", but comparison between this scheme and usual two-parent recombination was not performed.

An interesting attempt to combine genetic algorithms with the Simplex Method resulted in the ternary *simplex crossover* (Bersini and Seront 1992) . If x^1, x^2, x^3 are the three parents sorted in decreasing order of fitness, then the simplex crossover generates one child x by the following two rules.

(i) If $x_i^1 = x_i^2$ then $x_i = x_i^1$;
(ii) if $x_i^1 \neq x_i^2$ then $x_i = x_i^3$ with probability p and $x_i = 1 - x_i^3$ with probability $1 - p$.

Using the value $p = 0.8$, the simplex GA performed better than the standard GA on the DeJong functions. The authors remark that applying a modified crossover on more than three parents "is worth to try".

The problem of placing actuators on space structures is addressed by Furuya and Haftka (1993). The authors compare different crossovers, among others they use uniform crossover with two as well as with three parents in a GA using integer representation. Based on the experimental results they conclude that the use of three parents did not improve the performance. This might be related to another conclusion, indicating that for this problem mutation is an efficient operator and crossover might not be important. Uniform crossover with an arbitrary number of parents is also used by Aizawa (1994) as part of a special schema sampling procedure in a GA, but the multi-parent feature is only a side-effect and is not investigated.

A so-called *triadic crossover* is introduced and tested by Pál (1994) for a multimodal spin-lattice problem. The triadic crossover is defined in terms of two parents and one extra individual, chosen randomly from the population. The operator creates one child; it takes the bits in positions where the first parent and the third individual have identical bits from this parent and the rest of the bits from the other parent. Clearly, the result is identical to the outcome of a voting crossover on these three individuals as parents. Although the paper is primarily concerned with different selection schemes, a comparison between triadic, 1-point and uniform crossover is made, where triadic crossover turned out to deliver the best results.

C3.7.3 Operators with undefined arity

In the introduction to this section we defined the arity of a recombination operator as the number parents it uses. In some cases this number depends on the outcomes of random drawings; the operator is called without knowing in advance how many parents would be applied. In this section we treat three mechanisms of this kind.

Global recombination in evolution strategies allows the use of more than two recombinants (Bäck 1996), (Schwefel 1995). In ES there are two basic types of recombination, intermediate and discrete recombination, both having a standard two-parent variant and a global variant. Given a population of μ individuals global recombination creates one offspring x by the following mechanism.

$$x_i = \begin{cases} x_i^{S_i} \text{ or } x_i^{T_i} & \textit{global discrete recombination} \\ x_i^{S_i} + \chi_i \cdot (x_i^{T_i} - x_i^{S_i}) & \textit{global intermediate recombination} \end{cases}$$

where the two parents x^{S_i}, x^{T_i} $(S_i, T_i \in \{1, \ldots, \mu\})$ are redrawn for each i anew and so is the contraction factor χ_i. The above definition applies to the object variables as well as the strategy parameter part, i.e. for the mutation stepsizes (σ's) and the rotation angles (α's). Observe that the multi-parent character of global recombination is the consequence of redrawing the parents x^{S_i}, x^{T_i} for each coordinate i. Therefore, probably more than two individuals contribute to the offspring x, but their number is not defined in advance. It is clear that investigations on the effects of different numbers of parents on algorithm performance could not be performed in the traditional ES framework. The option of using multiple parents can be turned on or off, that is, global recombination can be used or not, but the arity of the recombination operator is not tunable. Experimental studies on global versus two-parent recombination are possible, but so far there are almost no experimental results available on this subject. In (Schwefel 1995) it is noted that "appreciable acceleration" is obtained by changing to bisexual from asexual scheme (i.e. adding recombination using two parents to the mutation-only algorithm), but only "slight further increase" is obtained when changing from bisexual to multisexual recombination (i.e. using global recombination instead of the two-parent variant). Recall the remark on the name p-sexual voting. The terms bisexual and multisexual are not appropriate either for the same reason, individuals have no gender or sex, and recombination can be applied to any combination of individuals.

Gene-pool recombination (GPR) was introduced by Mühlenbein and Voigt (1996) as a multi-parent recombination mechanism for discrete domains. It is defined as a generalization of two-parent recombination (TPR). Applying GPR is preceded by selecting a gene-pool consisting of would-be parents. Applying GPR the two parent alleles of an offspring are randomly chosen for each locus with replacement from the gene-pool and the offspring allele is computed "using any of the standard recombination schemes for TPR". Theoretical analysis on infinite populations shows that GPR is mathematically more tractable than TPR. If n stands for the number of variables (loci), then the evolution with proportional selection and GPR is fully described by n equations, while TPR needs 2^n equations for the genotypic frequencies. In practice GPR converges about 25% faster than TPR for ONEMAX. The authors conclude that GPR separates the identification and the search of promising areas of the search space better, besides it searches more reasonably than does TPR. In (Voigt and Mühlenbein 1995) GPR is extended to continuous domains by combining it with uniform fuzzy two-parent recombination (UFTPR) from Voigt *et al* (1995). The resulting uniform fuzzy gene pool recombination (UFGPR) outperforms UFTPR on the spherical function in terms of realized heritability, giving it a higher convergence speed. The convergence of UFGPR is shown to be about 25% higher than that of UFTPR.

A very particular mechanism is the *linkage evolving genetic operator* (LEGO) as defined in (Smith and Fogarty 1996). The mechanism is designed to detect and propagate blocks of corresponding genes of potentially varying length during the evolution. Punctuation marks in the chromosomes denote the beginning and the end of each block and more chromosomes with the appropriately positioned punctuation marks are considered as donors of a whole block during the creation of a child. Although the multi-parent feature is only a side-effect, LEGO is a mechanism where more than two parents can contribute to an offspring.

C3.7.4 Operators with tunable arity

Unary reproduction operators, such as mutation, are often called asexual, based on the biological analogies. Sexual reproduction traditionally amounts to two-parent recombination in EC, but the operators discussed in the previous section show that the sexual character of recombination can be intensified, in the sense that more than two parents can be recombined. Nevertheless, this intensification is not graded, the multi-parent option can be turned on or off, but the extent of sexuality (the number of parents) cannot be tuned. In this section we consider recombination operators that make sexuality a graded, rather than a Boolean, feature by having an arity that can vary. In other words, the operators we survey here are called with a certain number of parents as input, and this number can be modified by the user.

An early paper mentioning multi-parent recombination is that of Bremermann *et al* (1966) on solving linear equations. It presents the definition of three different multi-parent recombination mechanisms, called m-tuple mating. Given m binary parent vectors $x^1, \ldots, x^m$, the *majority mating* mechanism creates one offspring vector x by choosing

$$x_i = \begin{cases} 0 & \text{if half or more of the parents has } x_i^j = 0 \\ 1 & \text{otherwise} \end{cases}$$

Another mating mechanism for m binary parent vectors is called *mating by crossing over*. Describing it in contemporary terms, the mechanism works by selecting $m-1$ crossover points (identical in each parent) and then composing one child by selecting exactly one segment from each parent. The third operator is called *mating by averaging* and it is defined for vectors of continuous variables. Quite naturally, the child x of parents $x^1, \ldots, x^m$ is defined by

$$x_i = \sum_{j=1}^{m} \lambda_j x_i^j$$

where $\sum_{j=1}^{m} \lambda_j = 1$. Unfortunately, only very little is reported on the performance of these operators. It is remarked that using majority mating and mating by crossing over the results were somewhat inconclusive; no definite benefit was obtained. Using mating by averaging, however, led to "spectacular effects" within a linear programming scheme, but these effects are not specified.

Scanning crossover has been introduced as a generalization and extension of uniform crossover in GAs creating one child from r parents (Eiben 1991), (Eiben *et al* 1994). The name is based on the following general procedure scanning parents and thus building the child from left-to-right. Let $x^1, \ldots, x^r$ be the selected parents of length L and let x denote the child.

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procedure scanning:
begin
  INITIALIZE position markers as i_1 = ... = i_r := 1;  % mark 1st position in each parent
  for i = 1 to i = L
    CHOOSE j ∈ {1,...,r};
    x_i := x^j_{i_j};                      % i-th allele of x is the i_j-th allele of x^j
    UPDATE position markers i_1,...,i_r;
end
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The above procedure provides a general framework for a certain style of multi-parent recombination, where the precise execution, hence the exact definition of the operator, depends on the mechanisms to CHOOSE and to UPDATE. In the simplest case the UPDATE operation can shift the markers one position to the right, i.e. $i_j := i_j + 1,\ j \in \{1, \ldots, r\}$, can be used. This is appropriate for bit-strings, integer and floating-point representation. Scanning can also be easily adapted to order-based representation, where each individual is a permutation, if the UPDATE operation shifts to the first allele which is not in the child yet:

$$i_j := min\{\ k \mid k \geq i_j,\ x_k^j \notin \{x_1, \ldots, x_{i_j}\}\}, \quad j \in \{1, \ldots, r\}$$

Observe, that because of the term $k \geq i_j$ above, a marker can remain at the same position after an UPDATE, and will only be shifted if the allele standing at that position is included in the child. This guarantees that each offspring will be a permutation.

Depending on the mechanism to choose a parent (and thereby an allele) there are three different versions of scanning. The choice can be deterministic, choosing a parent containing the allele with the highest number of occurrences and breaking ties randomly (*occurrence based scanning*). Alternatively it can be random, either unbiased, following a uniform distribution thus giving each parent an equal chance to deliver its allele (*uniform scanning*), or biased by the fitness of the parents, where the chance of being chosen is fitness proportional (*fitness based scanning*). Uniform scanning for $r = 2$ is the same as uniform crossover, although creating only one child, and it also coincides with discrete recombination in evolution strategies. The occurrence based version is very much like the voting or majority mating mechanism discussed before, but without the threshold v, respectively with $v = \lfloor m/2 \rfloor$. The effect of the number of parents in scanning crossover has been studied in several papers. An overview of these studies is given in the next subsection.

Diagonal crossover has been introduced as a generalization of 1-point crossover in GAs (Eiben *et al* 1994). In its original form diagonal crossover creates r children from r parents by selecting $(r-1)$ crossover points in the parents and composing the children by taking the resulting in r chromosome segments from the parents 'along the diagonals'. Later on, a one-child version was introduced (van Kemenade *et al* 1995). Figure C3.7.1 illustrates both variants. It is easy to see that for $r = 2$ diagonal crossover coincides with 1-point crossover, and in some sense it also generalizes traditional 2-parent n-point crossover. To be precise, if we define (r, s)-*segmentation crossover* as working on r parents with s crossover points, diagonal crossover becomes its $(r, r-1)$ version, its $(2, n)$ variant coincides with n-point crossover and 1-point crossover is an instance of both schemes for $(r, s) = (2, 1)$ as parameters. The effect of operator arity for diagonal crossovers will be also discussed in the next subsection.

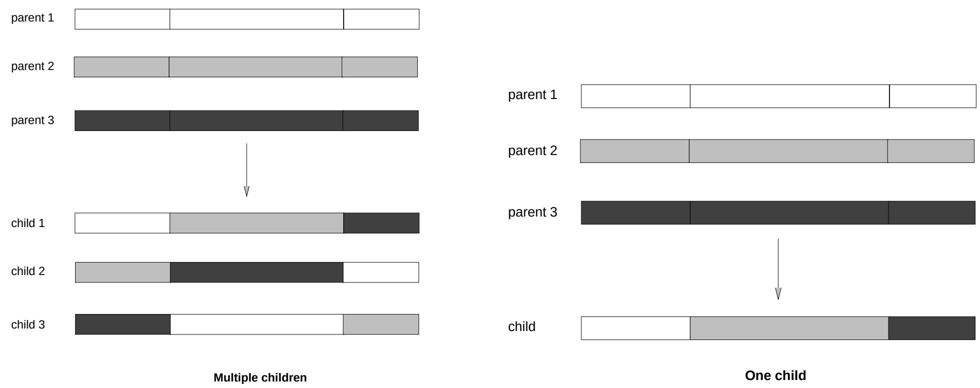

Figure C3.7.1. Diagonal crossover (left) and its one-child version (right) for 3 parents.

A recombination mechanism with tunable arity in ES is proposed by Schwefel and Rudolph (1995). The $(\mu, \kappa, \lambda, \rho)$-ES provides the possibility of freely adjusting the number of parents (called ancestors by the authors). The parameter ρ stands for the number of parents and global recombination is redefined for any given set $\{x^1, \ldots, x^\rho\}$ of parents as

$$x_i = \begin{cases} x_i^j & \rho\text{-ary discrete recombination} \\ \frac{1}{\rho} \cdot \sum_{k=1}^{\rho} x_i^k & \rho/\rho\text{-intermediate recombination} \end{cases}$$

where $j \in \{1, \ldots, \rho\}$ is uniform randomly chosen for each i independently. Let us note, that in the original paper, the above operators are called uniform crossover, respectively, global intermediate recombination. We introduce the names ρ-ary discrete recombination, respectively ρ/ρ-intermediate recombination here for the sake of a consequent terminology. (A reason for using the term ρ/ρ-intermediate recombination instead of ρ-ary intermediate recombination is given below, in the paragraph discussing the paper (Eiben and Bäck 1997).) Observe that ρ-ary discrete recombination coincides with uniform scanning crossover, while ρ/ρ-intermediate recombination is a special case of mating by averaging. At this time there are no experimental results available on the effect of ρ within this framework.

Related work in evolution strategies also uses ρ as the number of parents as an independent parameter for recombination (Beyer 1995). For purposes of a theoretical analysis it is assumed that all parents are different, uniform randomly chosen from the population of μ individuals. Beyer defines the ρ/ρ-intermediate recombination and ρ-ary discrete recombinations similarly to Schwefel and Rudolph (1995) and denotes them as intermediate (μ/ρ_I) recombination and dominant (μ/ρ_D) recombination, respectively. The $(\mu/\rho, \lambda)$ evolution strategy is studied on the spherical function for the special case of $\rho = \mu$. By this latter assumption it is not possible to draw conclusions on the effect of ρ, but the analysis shows that the optimal progress rate $\hat{\varphi}^*$ of the $(\mu/\mu, \lambda)$-ES is a factor μ higher than that of the (μ, λ)-ES, for both recombination mechanisms. Beyer hypothesizes that recombination has a statistical error correction effect, called genetic repair, and this effect can be improved by using more than two parents for creating offspring (Beyer 1996).

Another generalization of global intermediate recombination in evolution strategies is proposed by Eiben and Bäck (1997). The new operator is applied after selecting ρ parent individuals from the population of μ, and the resampling of two donors x^{S_i} and x^{T_i} for each i takes only these ρ individuals in consideration. Note, that this operator is also ρ-ary, just like the ρ/ρ-intermediate recombination as defined above, but utilizes only two donors for each allele of the offspring. To express this difference, this operator is called $\rho/2$-*intermediate recombination* and the operator of Beyer (1995) and Schwefel and Rudolph (1995) is called ρ/ρ-intermediate recombination. Observe, that the $\rho/2$-intermediate recombination is a true generalization of the original intermediate recombination: the case of $\rho = 2$ coincides with local intermediate recombination, while for $\rho = \mu$, it equals global intermediate recombination.

While intermediate recombination is based on taking the arithmetical average of the real-valued alleles of the parents, the geometrical avarage is computed by the *geometrical crossover*. Michalewicz *et al* (1996) present the definition for any ($k \geq 2$) number of parents, where the offspring of the parents $\{x^1, \ldots, x^k\}$ is defined as

$$x^{k+1} = \langle (x_1^1)^{\alpha_1} \cdot (x_1^2)^{\alpha_2} \cdot \ldots \cdot (x_1^k)^{\alpha_k}, \ldots, (x_n^1)^{\alpha_1} \cdot (x_n^2)^{\alpha_2} \cdot \ldots \cdot (x_n^k)^{\alpha_k} \rangle$$

where n is the chromosome length and $\alpha_1 + \ldots \alpha_k = 1$. The experimental part of the paper is, however, based on the two parent version, hence there are no results on the effect of using more than two parents with this operator.

The same holds for the so-called *sphere crossover* (Schoenauer and Michalewicz 1997); the authors give the general definition for k parents, but the experiments are restricted to the two parents version. In the general case the offspring of parents $\{x^1, \ldots, x^k\}$ is defined as

$$x^{k+1} = \langle \sqrt{\alpha_1 \cdot (x_1^1)^2 + \ldots + \alpha_k \cdot (x_1^k)^2}, \ldots, \sqrt{\alpha_1 \cdot (x_n^1)^2 + \ldots + \alpha_k \cdot (x_n^k)^2} \rangle$$

C3.7.5 The effects of higher operator arities

In the last years quite a few papers have studied the effect of operator arity on EA performance, some even in combination with varying selective pressure. Here we give a brief summary of these results, sorted by articles.

The performance of scanning crossover for different number of parents is studied in (Eiben *et al* 1994) in a generational GA with proportional selection. Bit-coded GAs for function optimization (DeJong functions F1 - F4 and a function from Michalewicz) as well as order-based GAs for graph coloring and the TSP are tested with different mechanisms to CHOOSE. In the bit-coded case more parents perform better than two, for the TSP and graph coloring 2 parents are advisable. Comparing different biases in choosing the child allele, on four out of the five numerical problems fitness based scanning outperforms the other two and occurrence based scanning is the worst operator.

In (Eiben *et al* 1995) diagonal crossover is investigated, compared to the classical 2-parent n-point crossover and uniform scanning in a steady-state GA with linear ranked biased selection ($b = 1.2$) and worst-fitness deletion. The test suite consists of two 2-dimensional problems (F2 and a function from Michalewicz) and four scalable functions (after Ackley, Griewangk, Rastrigin and Schwefel). The performance of diagonal crossover and n-point crossover shows a significant correspondence with r, respectively n. The best performance is always obtained with high values, between 10–15, where 15 was the maximum tested. Besides, diagonal crossover is always better than n-point crossover using the same number of crossover points ($r = n - 1$), thus representing the same level of disruptiveness. For scanning the relation between r and performance is less clear, although the best performance is achieved for more than two parents on five out of the six test functions.

The interaction between selection pressure and the parameters r for diagonal crossover, respectively n for n-point crossover is investigated in (van Kemenade *et al* 1995). A steady-state GA with tournament selection (tournament size between 1–6) combined with random deletion and worst-fitness deletion was applied to the Griewangk and the Schwefel functions. The disruptiveness of both operators increases parallely as the values for r and n are raised, but the experiments show that diagonal crossover consistently outperforms n-point crossover. The best option proves to be low selection pressure and high r in diagonal crossover combined with worst-fitness deletion.

Motivated by the difficulties to characterize the shapes of numerical objective functions, the effects of operator arity are studied on fitness landscapes with controllable ruggedness by Eiben and Schippers (1996). The NK-landscapes of Kauffman (1993), where the level of epistasis, hence the ruggedness of the landscape can be tuned by the parameter K, are used for this purpose. The multiple-children and the one-child version of diagonal crossover and uniform scanning are tested within a steady-state GA with linear ranked biased selection ($b = 1.2$) and worst-fitness deletion for $N = 100$ and different values of K. Two kinds of epistatic interactions, nearest neighbor interaction (NNI) and random neighbor interaction (RNI) are considered. Similarly to earlier findings (Eiben *et al* 1995), the tests show that the performance of uniform scanning cannot be related to the number of parents. The two versions of diagonal crossover behave identically, and for both operators there is a consequent improvement when increasing r. However, as the epistasis (ruggedness of the landscape) grows from $K = 1$ to $K = 5$ the advantage of more parents becomes smaller. On landscapes with significantly high epistasis ($K = 25$) the relationship between operator arity and algorithm performance seems to diminish. We illustrate these observations with a figure showing the error (deviation of the best individual from the optimum) at termination for the case of NNI in Figure C3.7.2. The final conclusions of this investigation can be very well related to works of (Schaffer and Eshelman 1991), (Eshelman and Schaffer 1993) and (Hordijk and Manderick 1995) on the usefulness of (2-parent) recombination. It seems that if and when crossover is useful, i.e. on mildly epistatic problems, then multi-parent crossover can be more useful than the 2-parent variants.

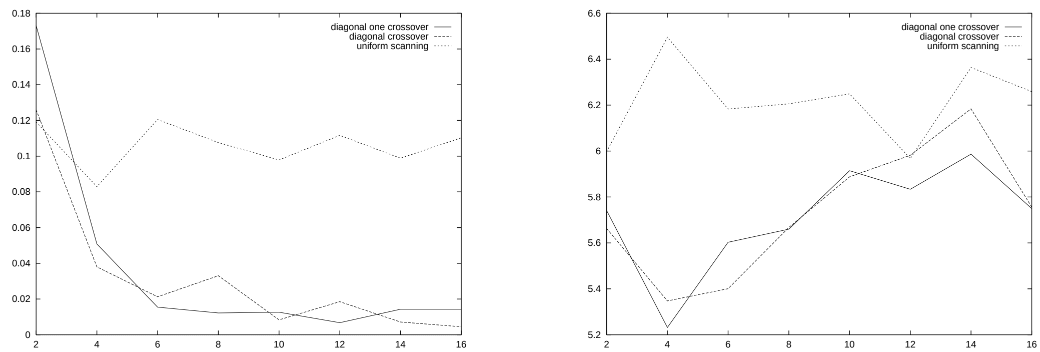

Figure C3.7.2. Illustration of the effect of the number of parents (horizontal axis) on the error at termination (vertical axis) on NK-landscapes with nearest neighbor interaction, $N = 100$, $K = 1$ (left) $K = 25$ (right).

The results of an extensive study of diagonal crossover for numerical optimization in GAs are reported in (Eiben and van Kemenade 1997). Diagonal crossover is compared to its one offspring version and n-point crossover on a test suite consisting of 8 functions, monitoring the speed, i.e., total number of evaluations, the accuracy, i.e., the median of the best objective function value found (all functions have an optimum of zero), and the success rate, i.e., the percentage of runs where the global optimum is found. In most of the cases an increase of performance can be achieved by increasing the disruptivity of the crossover operator (using higher values of n for n-point crossover), and even more improvement is achieved if the disruptivity of the crossover operator *and* the number of parents is increased (using more parents for diagonal crossver). This study gives a strong indication that for diagonal crossover an advantageous multi-parent effect does exist, that is, a) using this operator with more than two parents increases GA performance, and b) this improvement is not only the consequence of the increased number of crossover points.

A recent investigation of Eiben and Bäck (1997) addresses the working of multi-parent recombination operators in continuous search spaces, in particular within evolution strategies. This study compares $\rho/2$-intermediate recombination, ρ-ary discrete recombination, which is identical to uniform scanning crossover, and diagonal crossover with one child. Experiments are performed on unimodal landscapes (sphere model and Schwefel's double sum), multimodal functions with regularly arranged optima and a superimposed unimodal topology (Ackley, Griewangk and Rastrigin functions) and on the Fletcher-Powell and the Langermann functions that have an irregular, random arrangement of local optima. On the Fletcher-Powell function multi-parent recombination does not increase EA performance, besides on the unimodal double sum increasing operator arity decreases performance. Other conclusions seem to depend on the operator in question; the most consequent improvement for raising the number of parents is obtained for diagonal crossover.

C3.7.6 Conclusions

The idea of applying more than two parents for recombination in an evolutionary problem solver has occurred already in the sixties (Bremermann *et al* 1966). Several authors have designed and applied recombination operators with higher arities for a specific task, or used an existing operator with an arity higher than two (Kaufman 1967), (Mühlenbein 1989), (Bersini and Seront 1992), (Furuya and Haftka 1993), Aizawa (1994), (Pál 1994). Nevertheless, investigations explicitly devoted to the effect of operator arity on EA performance are still scarce; the study of the phenomenon of multi-parent recombination has just began. What would such a study mean? Similarly to the question whether binary reproduction operators (crossover with two parents) have advantages over unary ones (using mutation only), it can be investigated whether or not using more than two parents is advantageous. In case of operators with tunable arity this question can be refined and the relationship between operator arity and algorithm performance can be studied. It is, of course, questionable whether multi-parent recombination can be considered as one single phenomenon showing one behavioral pattern. The survey presented here discloses that there are (at least) three different types of multi-parent mechanisms with tunable arity.

(i) Operators based on allele frequencies among the parents, such as majority mating, voting recombination, ρ-ary discrete recombination, or scanning crossover.
(ii) Operators based on segmenting and recombining the parents, such as mating by crossing over, diagonal crossover or (r, s)-segmentation crossover.
(iii) Operators based on numerical operations, in particular averaging, of (real valued) alleles, such as mating by averaging, ρ/ρ-intermediate recombination, $\rho/2$-intermediate recombination, geometrical and sperical crossover.

A priori it cannot be expected that these different schemes show the same response to raising operator arities. There are also experimental results supporting differentiation among various multi-parent mechanisms. For instance, there seems to be no clear relationship between the number of parents and the performance of uniform scanning crossover, while the opposite is true for diagonal crossover (Eiben and Schippers 1996).

Another aspect multi-parent studies have to take into consideration is the expectedly different behavior on different types of fitness landscapes. As no single technique would work on every problem, multi-parent mechanisms will have their limitations too. Some studies indicate that on irregular landscapes, such as NK-landscapes with relatively high K values (Eiben and Schippers 1996), or the Fletcher-Powell function (Eiben and Bäck 1997) they do not work. On the other hand, on the same Fletcher-Powell function Eiben and van Kemenade (1997) observed an advantage of increasing the number of parents for diagonal crossover in a GA framework using bit-coding of variables, although, they also found indications that this can be an artifact, caused simply by the increased disruptiveness of the operator for higher arities. Investigations on multi-parent effects related to fitness landscape characteristics smoothly fit into the tradition of studying the (dis)advantages of two-parent crossovers under different circumstances, (Schaffer and Eshelman 1991), (Eshelman and Schaffer 1993), (Spears 1993), (Hordijk and Manderick 1995).

Let us also touch on the issue of practical difficulties when using multi-parent recombination operators. Introducing operator arity as a new parameter implies an obligation of setting its value. Since so far there are no reliable heuristics for setting this parameter, finding good values may require numerous tests, prior to 'real' application of the EA. A solution can be based on previous work on adapting (Davis 1989) or self-adapting (Spears 1995) the frequency of applying different operators. Alternatively, a number of competing subpopulations could be used in the spirit of (Schlierkamp-Voosen and Mühlenbein 1996). According to the latter approach each different arity is used within one subpopulation and subpopulations with greater progress, i.e. with more powerful operators, become larger. A first assessment of this technique can be found in (Eiben *et al* 1998).

Concluding this survey we can note the following. Even though there are no biological analogies of recombination mechanisms where more than two parent genotypes are mixed in one single recombination act, formally there is no necessity to restrict the arity of reproduction mechanisms to one (mutation) or two (crossover) in computer simulations. Studying the phenomenon of multi-parent recombination has just began, but there is already substantial evidence that applying more than two parents can increase the performance of EAs. Considering multi-parent recombination mechanisms is thus a sound design heuristics for practitioners and a challenge for theoretical analysis.

References

Ackley D 1987 An empirical study of bit vector function optimization *Genetic Algorithms and Simulated Annealing* ed. Davis L, pages 170–215 Morgan Kaufmann

Aizawa A N 1994 Evolving SSE: A Stochastic Schemata Exploiter *Proceedings of the 1st IEEE Conference on Evolutionary Computation*, pages 525–529 IEEE Press

Bäck T 1996 *Evolutionary Algorithms in Theory and Practice* Oxford University Press, New York

Bersini H and Seront G 1992 In search of a good evolution-optimization crossover *Proceedings of the 2nd Conference on Parallel Problem Solving from Nature* ed. Männer R and Manderick B, pages 479–488 North-Holland

Beyer H.-G 1995 Toward a theory of evolution strategies: On the benefits of sex- the $(\mu/\mu, \lambda)$ theory *Evolutionary Computation*, 3(1):81–111

Beyer H.-G Dagstuhl, Germany, 25–29 May 1996 Basic principles for a unified EA-theory *Handout of the talk given at the Evolutionary Algorithms and their Applications Workshop*

Bremermann H Rogson M and Salaff S 1966 Global properties of evolution processes *Natural Automata and Useful Simulations* ed. Pattee H Edlsack E Fein L and Callahan A, pages 3–41 Spartan Books, Washington DC

Davis L 1989 Adapting operator probabilities in genetic algorithms *Proceedings of the 3rd International Conference on Genetic Algorithms* ed. Schaffer J, pages 61–69 Morgan Kaufmann

Eiben A *A Method for Designing Decision Support Systems for Operational Planning* PhD Thesis Eindhoven University of Technology 1991

Eiben A and Bäck T 1997 An empirical investigation of multi-parent recombination operators in evolution strategies *Evolutionary Computation*, 5(3):347–365

Eiben A and Kemenade C. v 1997 Diagonal crossover in genetic algorithms for numerical optimization *Journal of Control and Cybernetics*, 26(3):447–465

Eiben A Kemenade C. v and Kok J 1995 Orgy in the computer: Multi-parent reproduction in genetic algorithms *Advances in Artificial Life. Third International Conference on Artificial Life* ed. Morán F Moreno A Merelo J. J and Chacón P, volume 929 of *Lecture Notes in Artificial Intelligence*, pages 934–945 Springer, Berlin

Eiben A Raué P.-E and Ruttkay Z 1994 Genetic algorithms with multi-parent recombination *Proceedings of the 3rd Conference on Parallel Problem Solving from Nature* ed. Davidor Y Schwefel H.-P and Männer R, number 866 in Lecture Notes in Computer Science, pages 78–87 Springer-Verlag

Eiben A and Schippers C 1996 Multi-parent's niche: n-ary crossovers on NK-landscapes *Proceedings of the 4th Conference on Parallel Problem Solving from Nature* ed. Voigt H.-M Ebeling W Rechenberg I and Schwefel H.-P, number 1141 in Lecture Notes in Computer Science, pages 319–328 Springer, Berlin

Eiben A Sprinkhuizen-Kuyper I and Thijssen B 1998 Competing crossovers in an adaptive GA framework *Proceedings of the 5th IEEE Conference on Evolutionary Computation* IEEE Press to appear

Eshelman L and Schaffer J 1993 Crossover's niche *Proceedings of the 5th International Conference on Genetic Algorithms* ed. Forrest S, pages 9–14 Morgan Kaufmann

Furuya H and Haftka R 1993 Genetic algorithms for placing actuators on space structures *Proceedings of the 5th International Conference on Genetic Algorithms* ed. Forrest S, pages 536–542 Morgan Kaufmann

Hordijk W and Manderick B 1995 The usefulness of recombination *Advances in Artificial Life. Third International Conference on Artificial Life* ed. Morán F Moreno A Merelo J. J and Chacón P, volume 929 of *Lecture Notes in Artificial Intelligence*, pages 908–919 Springer, Berlin

Kauffman S 1993 *Origins of Order: Self-Organization and Selection in Evolution* Oxford University Press, New York, NY

Kaufman H 1967 An experimental investigation of process identification by competitive evolution *IEEE Transactions on Systems Science and Cybernetics*, SSC-3(1):11–16

, Michalewicz Z Nazhiyath G and Michalewicz M 1996 A Note on Usefulness of Geometrical Crossover for Numerical Optimization Problems *Proceedings of the 5th Annual Conference on Evolutionary Programming* ed. Fogel L J Angeline P J and Bäck Th, pages 305–311, MIT Press

Mühlenbein H 1989 Parallel genetic algorithms, population genetics and combinatorial optimization *Proceedings of the 3rd International Conference on Genetic Algorithms* ed. Schaffer J, pages 416–421 Morgan Kaufmann

Mühlenbein H and Voigt H.-M 1996 Gene pool recombination in genetic algorithms *Meta-Heuristics: Theory and Applications* ed. Osman I and Kelly J, pages 53–62, Boston, London, Dordrecht Kluwer Academic Publishers

Pál K 1994 Selection schemes with spatial isolation for genetic optimization *Proceedings of the 3rd Conference on Parallel Problem Solving from Nature* ed. Davidor Y Schwefel H.-P and Männer R, number 866 in Lecture Notes in Computer Science, pages 170–179 Springer-Verlag

Schaffer J and Eshelman L 1991 On crossover as an evolutionary viable strategy *Proceedings of the 4th International Conference on Genetic Algorithms* ed. Belew R and Booker L, pages 61–68 Morgan Kaufmann

Schlierkamp-Voosen D and Mühlenbein H 1996 Adaptation of population sizes by competing subpopulations *Proceedings of the 3rd IEEE Conference on Evolutionary Computation*, pages 330–335 IEEE Press

Schwefel H.-P 1995 *Evolution and Optimum Seeking* Wiley, New York

Schwefel H.-P and Rudolph G 1995 Contemporary evolution strategies *Advances in Artificial Life. Third International Conference on Artificial Life* ed. Morán F Moreno A Merelo J. J and Chacón P, volume 929 of *Lecture Notes in Artificial Intelligence*, pages 893–907 Springer, Berlin

Schoenauer M and Michalewicz Z 1997 Boundary Operators for Constrained Parameter Optimization Problems *Proceedings of the 7th International Conference on Genetic Algorithms* ed. Bäck Th, pages 322–329 Morgan Kaufmann

Smith J and Fogarty T 1996 Recombination strategy adaptation via evolution of gene linkage *Proceedings of the 3rd IEEE Conference on Evolutionary Computation*, pages 826–831 IEEE Press

Spears W 1993 Crossover or mutation? *Foundations of Genetic Algorithms - 2* ed. Whitley L, pages 221–238 Morgan Kaufmann

Spears W 1995 Adapting crossover in evolutionary algorithms *Proceedings of the 4th Annual Conference on Evolutionary Programming* ed. McDonnell J Reynolds R and Fogel D, pages 367–384 MIT Press

van Kemenade C Kok J and Eiben A 1995 Raising GA performance by simultaneous tuning of selective pressure and recombination disruptiveness *Proceedings of the 2nd IEEE Conference on Evolutionary Computation*, pages 346–351 IEEE Press

Voigt H.-M and Mühlenbein H 1995 Gene pool recombination and utilization of covariances for the Breeder Genetic Algorithm *Proceedings of the 2nd IEEE Conference on Evolutionary Computation*, pages 172–177 IEEE Press

Voigt H.-M Mühlenbein H and Cvetković D 1995 Fuzzy recombination for the Breeder Genetic Algorithm *Proceedings of the 6th International Conference on Genetic Algorithms* ed. Forrest S, pages 104–111 Morgan Kaufmann

