1. What is the type of the polymorphic identity?

2. Show how the polymorphic identity is used to get the identity on the type \texttt{nat} of natural numbers.

3. Give the polymorphic version of the following function:
\[
\lambda f : \texttt{nat} \to \texttt{bool} \to \texttt{nat}. \lambda x : \texttt{nat}. \lambda y : \texttt{bool}. f x y.
\]
(In the polymorphic variant neither \texttt{nat} nor \texttt{bool} occurs.)

4. Explain why the following proof is not correct:
\[
\begin{array}{c}
\exists a. a \to b \\
\frac{[a \to b^\forall]}{(a \to b) \to (a \to b)} \\
\frac{I[x]}{a \to b} \\
E\exists \\
\end{array}
\]

5. Show that \( \forall a. (\forall b. b) \to a \) is a tautology.

6. Give the \( \lambda \text{2-term} \) corresponding to the formula \( \forall a. (\forall b. b) \to a \).

7. Give a \( \lambda \text{2-term} \) that is an inhabitant of the answer to the previous exercise.

8. Show that \( (\forall c. ((a \to b \to c) \to c)) \to a \) is a tautology of second-order minimal propositional logic.

9. What is the impredicative definition of \( \bot \) in second-order propositional logic?

10. (Following the previous exercise.) What is the corresponding term in \( \lambda \text{2} \)?
11. Define the type \texttt{new\_or}

\[
(new\_or\ A\ B) = \Pi c: \ast . (A \to c) \to (B \to c) \to c
\]

Assume \(\Gamma \vdash a : A\). Give an inhabitant of \((new\_or\ A\ B)\).
(NB: it is not asked to give the type derivation.)

12. Assume \texttt{new\_or} as in the previous exercise, and in addition \(\Gamma \vdash f : A \to D\), and \(\Gamma \vdash g : B \to D\), and \(\Gamma \vdash M : (new\_or\ A\ B)\). Give an inhabitant of \(D\).
(NB: it is not asked to give the type derivation.)

13. We define the booleans \texttt{B} and \texttt{true} (\texttt{T}) and \texttt{false} (\texttt{F}) as follows:

\[
\begin{align*}
B &= \Pi a: \ast . a \to a \to a \\
T &= \lambda a: \ast . \lambda x:a. \lambda y:a. x \\
F &= \lambda a: \ast . \lambda x:a. \lambda y:a. y
\end{align*}
\]

Give a definition of negation in \(\lambda 2\).

14. We assume \(a : \ast\). Give inhabitants in \(\lambda 2\) of the following types:

(a) \((\Pi b : \ast . b) \to a\),
(b) \(a \to \Pi b : \ast . (b \to a)\),
(c) \(a \to \Pi b : \ast . ((a \to b) \to b)\).