Third exercise

Given is that

$$\mathbb{E}_{\mathbb{Q}}(B_T^{-1}X) = \frac{1}{\sqrt{2\pi}} \int_A^\infty \left(S_0 e^{-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}x} - E e^{-rT} \right) e^{-\frac{x^2}{2}}\, dx,$$

where

$$A = \frac{(\frac{1}{2}\sigma^2 - r)T - \log\left(\frac{S_0}{E}\right)}{\sigma\sqrt{T}}.$$

Prove that this is equal to

$$S_0 \Phi(d_1) - E e^{-rT} \Phi(d_2),$$

where

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{S_0}{E}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \\ d_2 &= \frac{\log\left(\frac{S_0}{E}\right) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \end{aligned}$$

and

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-\frac{x^2}{2}}\, dx.$$