# Studyguide Mathematical System Theory

## Introduction

The course Mathematical Systems Theory is based on the book "Mathematical Systems Theory", written by Dr. Chr. Heij, Prof. dr. A.C.M. Ran and Dr. F. van Schagen. The course consists of lectures and a series of assignments. The latter are called the "practicum". To support you in the practicum there is an office hour. You may also address your questions to A.C.M. Ran at other hours. During the lectures the theory will be explained, and some applications will be given. The assignments are meant to help you understand the theory, and to give you the opportunity to show you master the material. The practicum will make extensive use of Matlab.

## Lecturers and times

The lectures will be given by J.H. van Schuppen according to the following schedule

| Lecture | Week | Date | Day | Hours | Room |
|---|---|---|---|---|---|
| 01 | 05 | Feb. 05 | Friday | 10:00-12:45 | S 640 |
| 02 | 06 | Feb. 12 | Friday | 10:00-12:45 | S 640 |
| 03 | 07 | Feb. 19 | Friday | 10:00-12:45 | S 640 |
| 04 | 09 | Mar. 05 | Friday | 10:00-12:45 | S 640 |
| 05 | 10 | Mar. 12 | Friday | 10:00-12:45 | S 640 |
| 06 | 11 | Mar. 19 | Friday | 10:00-12:45 | S 640 |
| 07 | 13 | Mar. 29 | Monday | 13:30-16:15 | C 668 |
| 08 | 14 | Apr. 09 | Friday | 10:00-12:45 | C 624 |
| 09 | 15 | Apr. 16 | Friday | 10:00-12:45 | S 624 |
| 10 | 16 | Apr. 23 | Friday | 10:00-12:45 | S 624 |
| 11 | 17 | Apr. 26 | Monday | 13:30-16:15 | C 668 |
| 12 | 18 | May 07 | Friday | 10:00-12:45 | S 624 |
| 13 | 20 | May 21 | Friday | 10:00-12:45 | S 624 |

Besides there are office hours: Fridays 08:30-09:45, in T 01.68

| name | room | telephone | e-mail |
|---|---|---|---|
| J.H. van Schuppen | T 01.68b | | J.H.vanSchuppen@cs.vu.nl |
| A.C.M. Ran | R 345 | 5987691 | ran@cs.vu.nl |

Van Schuppen's url is:
http://www.cwi.nl/∼schuppen
There you may also find the website for this course.

## Lectures

During the lectures the theory will be summarized and examples given. This is also the time and place to ask questions about the theory. We expect the students to study the theory on time, so at the latest in the week following the lecture.

## Examination and grading

The course is graded based partly on the practicum assignments that are handed in, and partly on an oral examination at the end of the course. Roughly, the practicum will count for about 7/10, the oral examination for about 3/10.

## Practicum

The assignments have to be done in pairs. It is the intention that a pair should work together so that each student does his share of the work, but understands the whole. Both students in a pair are responsible for the final result. Exchange of information between students from different pairs is allowed, but copying work from others (also from students who took the course in previous years) is considered fraud, and will be punished. To repeat: it is allowed to get essential information from a fellow student or one of the teachers, but the work you hand in should be your own.

## Question hour

The one-hour lectures will frequently be used to answer question about the assignments, and to give hints for the solutions. Also, on occasion some material that we consider pre-requisite material (i.e., from linear algebra, analysis or statistics) will be treated in the question hour. This is also the time and place to profit from questions others may have, and to exchange tips about the efficient and elegant use of matlab.

Further down in this study guide you will get information about the demands on the work to be turned in. An example will be given of what we expect.

## Office hours

The office hours are meant to discuss individual problems you encounter while making the assignments. It is **not** the intention of the office hours that this is the time and place to answer questions on the theory to students who have not been to the lectures. A private lecture will not be given. It is also not the intention that the lecturer serves as your matlab debugger.

## Books, Matlab help

For linear algebra we use a standard source: the book by D. Lay 'Linear Algebra and its Applications'. For systems theory, obviously the book is the source of information. Matlab has extensive 'on-line' help. The commands 'help' or 'helpwin' will lead you to a lot of information in an efficient way. Using 'helpdesk' a complete manual is online available. The file in our system is
/usr/local/matlab/help/helpdesk.html

The assignments and the data you need for them are available either on the CD rom that accompanies the book.

Before the command 'matlab' will work (under unix) you first have to set the path-variable in the file '.profile' in your home-directory. Insert, directly after PATH= the phrase '/usr/local/matlab/bin:' . The colon is the separator with the next directory in the path. In any case /usr/local/matlab/bin should precede /usr/local.

There is a Student Edition of Matlab, which will run on your laptop. It is not cheap but if you have several courses requiring the use of Matlab you may consider it. Most of the exercises will actually only require older versions of what is in the Student Edition (e.g., the Matlab 5 student edition will still solve most, if not all of the exercises). Such older versions may be available at used-book stores.

## Due dates and grading of assignments

In the table below you will find the due dates for the series of exercises. The last set consists of two parts. The dates mentioned are Saturdays, which may be interpreted as Monday before nine in the morning.

| serie | datum |
|---|---|
| 1 | 20/2/2010 |
| 2 | 13/3/2010 |
| 3 | 27/3/2010 |
| 4 | 17/4/2010 |
| 5a | 8/5/2010 |
| 5b | 29/5/2010 |

Each of the series is graded separately. In principal an insufficient grade means you have to re-do the series, we do allow for one 5. The final grade for the practicum is the average of the grades. It is essential to hand in your assignments on time. Should the dates be inconvenient on account of other courses for which the whole group (or a large part of it) has to hand in homework, then we can shift a bit.

## What and how to hand in your work.

The first practicum consists of a number of exercises and a number of assignments. You have to hand in the assignments. As a rule, the other series only consist of exercises which you have to hand in (with one exception). In all the series of exercises/assignments, we expect per exercise you hand in on paper a solution that consists of the following separate items:

*Conclusions.* A summary of the problem with the answers. This must be clear, readable and understandable without knowledge of Matlab. You have to explain what the answer means. You can hand this in on paper in the mailbox of Ran, or, if you absolutely cannot do it any other way, by email (ran@cs.vu.nl). However, in that case only pdf or latex files are considered. Doc-files are never considered a good way of handing in your work.

*Method of solution.* This is a description of the method of solution you used, with a readable, mathematically correct and transparent justification. This must again be handed in on paper. If you so wish, you may combine this part with the conclusions to one piece, provided the conclusions are clearly formulated and regconizable as such in the text.

*Matlab-documentation.* A complete documentation of a Matlab-session that leads from the given data to the answer. (Obviously, this does not hold for the more theoretical exercises.) You may use the Matlab command

'diary' for this purpose (see 'help diary' in Matlab). Please make sure that you clean up your Matlab session a bit before handing this in. We are not interested in all the wrong turns you took in the process. Note: this part **has to be turned in electronically, so by email, to ran@cs.vu.nl**. What you hand in has to show clearly what the commands where that you used, and what the relevant reaction of Matlab was. Obviously, you should refrain from sending us large amounts of numbers. We don't want to see a 100 by 100 matrix displayed on screen, and neither would you. Likewise, neither we nor you are interested in all the intermediate results of a long for-loop. Remember that by using ';' at the end of a line you suppress Matlab's output. Also "diary on" and "diary off" are good ways of omitting output in the diary file.

The most important reason why we want this electronically is that we can reconstruct what you did by running the code ourselves. So, beware of editing diary files. Make sure the resulting file is still something that produces the right output at the right place.

**Example.**
*Exercise.* Let

$$A = \frac{1}{15}\begin{bmatrix} -283 & 176 & 200 \\ 36 & 33 & 0 \\ -452 & 244 & 325 \end{bmatrix}.$$

Determine a number $a$ and a vector $v \neq 0$ such that $\lim_{n\to\infty} a^{-n}A^n x = \alpha(x)v$ for every vector $x$. Here it is required that $\alpha(x)$ is a function of $x$ such that $\alpha(v) \neq 0$.

*1. Solution.* The number $a$ is 7 and for the vector $v$ we choose (in 4 decimals after the decimal point)

$$v = \begin{bmatrix} 0.5345 \\ 0.2673 \\ 0.8018 \end{bmatrix}.$$

*2. Solution method.* We determine an invertible matrix $V$ and a diagonal matrix $D$ such that $A = VDV^{-1}$. For the given $A$ this is possible. The matrix $D$ has the eigenvalues of $A$ on its diagonal. The number $a$ is chosen as the eigenvalue for which the absolute value is greater than that of the other eigenvalues of $A$. In general this may not be unique, but in this case it is. Without loss of generality that greatest value is the absolute value of

the eigenvalue $d_{33}$. For $v$ we choose the corresponding eigenvector. Up to a multiplicative factor this is unique. Note that the columns of $V$ consist of eigenvectors of $A$. More precisely, the $j$'th column of $V$ is an eigenvector corresponding to the $j$'th element on the diagonal of $D$. So, we choose $v = v_3$, the third column of $V$. Now we have to show that $\lim_{n\to\infty} a^{-n}A^n x = \alpha(x)v$ for every vector $x$. Let $y = V^{-1}x$.Then, with $v_1$, $v_2$ and $v_3$ being the columns of $V$, we find

$$a^{-n}A^n x = a^{-n}VD^n y = \left(\frac{d_{11}}{a}\right)^n y_1 v_1 + \left(\frac{d_{22}}{a}\right)^n y_2 v_2 + y_3 v_3.$$

Since $|d_{11}| < |a|$ and $|d_{22}| < |a|$ we see that $\lim_{n\to\infty} a^{-n}A^n x = y_3 v_3 = y_3 v$. Now choose $\alpha(x) = y_3$ and note that $\alpha(v) = 1$. So, the number $a$ has the desired properties.

*3. Matlab-session.* (Here on paper, by we expect it as a file.)

```
⟩⟩ format compact    % this makes sure there are not too many empty lines
⟩⟩ A = (1/15) * [-283 176 200 ; 36 33 0 ; -452 244 325]      % type in A.
A =
 −18.8667   11.7333   13.3333
   2.4000    2.2000         0
 −30.1333   16.2667   21.6667
⟩⟩ [V,D]=eig(A) % Determine the eigenvalues and eigenvectors of A.
V =
 −0.5883   −0.3015   −0.5345
  0.1961   −0.9045   −0.2673
 −0.7845    0.3015   −0.8018
D =
 −5.0000         0         0
       0    3.0000         0
       0         0    7.0000
⟩⟩ a=D(3,3)
a =
7.0000
⟩⟩ v=V(:,3) % Select the third column.
v =
-0.5345
-0.2673
-0.8018
```