DISTRIBUTED ALGORITHMS 2014 SEMINAR

Anastasija Efremovska and István Haller

Vrije Universiteit Amsterdam

November 19, 2014
Seminar 7

- 10 exercises -
Exercise 1

Give a centralized algorithm for assigning unique IDs to processes that terminates after at most $D + 1$ time units. ($D$ is the network diameter)

Reminder

Centralized Identifier Assignment Algorithm

- **goal** - assign unique identifiers to all participating processes
- **centralized** - there is a known single initiator
EXERCISE 1

Give a centralized algorithm for assigning unique IDs to processes that terminates after at most \( D + 1 \) time units. (\( D \) is the network diameter)

REMINDER

Centralized Identifier Assignment Algorithm

- **goal** - assign unique identifiers to all participating processes
- **centralized** - there is a known single initiator
Assume a Monte Carlo algorithm, and a (deterministic) algorithm to check whether the Monte Carlo algorithm terminated correctly. Give a Las Vegas algorithm that terminates with probability one. Suppose the Monte Carlo algorithm gives a correct outcome with some probability $\pi$. How many applications of this algorithm does it take on average to come to a correct outcome?
REMINDER

Probabilistic Algorithms

**Monte Carlo**
- it always terminates
- the probability that a terminal configuration is correct is greater than zero

**Las Vegas**
- the probability that it terminates is greater than zero
- all terminal configurations are correct
Exercise 3

Apply the Itai-Rodeh election algorithm to an anonymous directed ring of size three, in which all processes know the network size. Initially, let two processes select the ID $i$, and one the ID $j$, with $i > j$. Give one possible computation.
SEMANTIC 7 - EXERCISE 3

**REMINDER**

Itai-Rodeh Election Algorithm

- **G** is an *anonymous, directed* ring; all processes know the ring size **N**
- Each initiator selects a random ID from \{1; \cdots ; N\}
- Each process sends out an ID, and the **largest** ID is the only one making a round trip
- *Care!* different processes may select the same ID
- Each message is supplied with a *hop count*; a message arrives at its source if and only if its hop count is **N**
- If several processes select the same smallest ID, then they will start a fresh election round, at a *higher level*
- *Passive* processes pass on messages, with inc in hop count
Apply the **echo algorithm with extinction** to elect a leader in the following anonymous undirected network. All processes are initiators and know the network size. In election round 0, let p and r select ID i, while q and s select ID j, with \( i > j \). Give a computation in which s becomes the leader in round 1. Explain why, in such a computation, p and r will not both progress to round 1.
**REMINDER**

**Echo algorithm with extinction**

- all processes know the *network size*
- initially, initiators are *active* at level 0, and non-initiators are *passive*
- each active process selects a random ID, and starts a wave, tagged with its ID and level 0
- suppose process $p$ in wave $v$ at level $l$ is hit by wave $w$ at level $l'$:
  1. if $l < l'$, or $l = l'$ and $v > w$, then $p$ changes to wave $w$ at level $l'$, and treats the message according to the echo algorithm
  2. if $l > l'$, or $l = l'$ and $v < w$, then $p$ purges the message
  3. if $l = l'$ and $v = w$, then $p$ treats the message according to the echo algorithm
Reminder

Echo algorithm with extinction

- Each message sent upwards in the constructed tree reports the size of its subtree. All other messages report 0.
- When a process *decides*, it computes the size of the constructed tree.
- If the constructed tree covers the network, it becomes the leader.
- Otherwise, it selects a new id, and initiates a new wave, at a higher level.
EXERCISE 6
Argue that there is no Las Vegas algorithm for election in anonymous rings of unknown size.

HINT
Theorem: There is no Las Vegas algorithm to compute the size of an anonymous ring.
Exercise 7

Give a Monte Carlo algorithm for election in anonymous networks of unknown size. What is the success probability of your algorithm?

Hint

Algorithm selection up to you.
EXERCISE 9

Give an (always correctly terminating) algorithm for computing the size of anonymous acyclic networks.

HINT

Think about an algorithm we have been using quite frequently of an anonymous ring.
Exercise 10

Apply the Itai-Rodeh ring size algorithm to an anonymous directed ring of size 3 in the following two cases:

(A) Suppose that all three processes initially choose the same ID. Show that the algorithm computes ring size 2.

(B) Suppose that only two processes initially choose the same ID. Show that the algorithm computes ring size 3.
**SEMINEAR 5 - EXERCISE 10**

**REMINDER**

Itai-Rodeh Ring Size Algorithm

- each process $p$ maintains an estimate $est_p$ of the ring size; initially $est_p = 2$ (always $est_p \leq N$)
- $p$ initiates an estimate round (1) at the start of the algorithm, and (2) at each update of $est_p$
- each round, $p$ selects a random $id_p$ in $\{1 \ldots R\}$ sends $(est_p; id_p; 1)$, and waits for a message $(est; id; h)$ (always $h \leq est$)

1. if $est < est_p$, $p$ purges the message
2. let $est > est_p$
   - if $h < est$, then $p$ sends $(est; id; h+1)$, and $est_p := est$
   - if $h = est$, then $est_p := est + 1$
3. let $est = est_p$
   - if $h < est$, then $p$ sends $(est; id; h+1)$
   - if $h = est$ and $id \neq id_p$, then $est_p := est + 1$
   - if $h = est$ and $id = id_p$, then $p$ purges the message (possibly its own message returned)