1. (a) No, because $(0.7)^2 + (0.3)^2 = 0.58 \neq 1$.
(b) Yes, because $(0.8)^2 + (0.6)^2 = 1$.
(c) Yes, because $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 = 1$.
(d) This is only a superposition if $\theta = \frac{k\pi}{2}$, because only then $|\cos \theta|^4 + |\sin \theta|^4 = 1$.

For (b) the probability to measure 0 is $(0.8)^2 = 0.64$.

For (c) the probability to measure 0 is $(0.5)^2 = 0.25$.

For (d) with $\theta = \frac{k\pi}{2}$ the probability to measure 0 is 1 if $k$ is even and 0 if $k$ is odd.

2. $\frac{3}{5} |0\rangle + \frac{4}{5} |1\rangle$

3. (a) 0.5
(b) 0.5
(c) 0

4. We apply a rotation of $-\frac{\pi}{8}$ to the second qubit:

$$
\left( \begin{array}{cc}
\cos \frac{\pi}{8} & \sin \frac{\pi}{8} \\
-\sin \frac{\pi}{8} & \cos \frac{\pi}{8}
\end{array} \right)
$$

The superposition of the 2-qubit then becomes

$$
\frac{1}{\sqrt{2}} (\cos \frac{\pi}{8} |00\rangle - \sin \frac{\pi}{8} |01\rangle + \sin \frac{\pi}{8} |10\rangle + \cos \frac{\pi}{8} |11\rangle)
$$

We no apply a rotation of $\frac{\pi}{8}$ to the first qubit:

$$
\left( \begin{array}{cc}
\cos \frac{\pi}{8} & -\sin \frac{\pi}{8} \\
\sin \frac{\pi}{8} & \cos \frac{\pi}{8}
\end{array} \right)
$$

The superposition of the 2-qubit then becomes

$$
\frac{1}{\sqrt{2}} ((\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}) |00\rangle - 2 \cdot \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} |01\rangle \\
+ 2 \cdot \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} |10\rangle + (\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}) |11\rangle)
$$

1
5. $y = 1$, so Bob applies a rotation of $-\frac{\pi}{8}$ to its qubit. The resulting superposition is

$$\frac{1}{\sqrt{2}} \left( \cos \frac{\pi}{8} |00\rangle - \sin \frac{\pi}{8} |01\rangle + \sin \frac{\pi}{8} |10\rangle + \cos \frac{\pi}{8} |11\rangle \right)$$

So $a \oplus b = 0$ with probability $\cos^2 \frac{\pi}{8}$. 