1. If $\mathcal{H}$ doesn’t reach a halt state on $(M, w)$ (which implies that $M$ halts on $w$), we will never start the execution of $M$ on $w$ to check whether $M$ accepts $w$.

2. (a) Yes. Both $w_3w_4w_1$ and $v_3v_4v_1$ are equal to 11101001.

   (b) No. The second symbol of $w_1 = 001$ is 0, while the second symbol of $v_1 = 01$ is 1.

   (c) $y_0 = \$0\$0\$1$ $y_1 = \$0\$0\$1$ $y_2 = \$0\$0\$1\$1$ $y_3 = \$1\$1$ $y_4 = \$0\$0\$1$ $y_5 = \#$

   $z_0 = \$0\$1$ $z_1 = \$0\$1$ $z_2 = \$1\$1\$1$ $z_3 = \$1\$1\$1$ $z_4 = \$0\$1\$0$ $z_5 = \$#$

3. (a) $(a + b)^*bb(a + b)^*$.

   (b) $\lambda$ $\#q_0abb$ $q_f\lambda$ $q_f$

   $a$ $a$ $q_a b$ $q_f$

   $b$ $b$ $q_f \square$ $q_f$

   $\square$ $\square$ $aq_f b$ $q_f$

   $\#$ $\#$ $b q_f a$ $q_f$

   $q_0 a$ $a q_0$ $\square q_f a$ $q_f$

   $q_0 b$ $b q_1$ $\# q_f a$ $\lambda$

   $q_1 a$ $a q_0$ $a q_1 b$ $q_f a b$

   $a q_1 b$ $a q_1 b$ $q_f a b$

   $b q_1 b$ $b q_f b b$

   $\square q_1 b$ $q_f \square b$

   $\# q_1 b$ $\# q_f \square b$

   (c) Start on the leftmost element of the string, in state $q_0$.

   Read $a$, leave it $a$, make one step to the right, and stay in state $q_0$.

   Read $b$, leave it $b$, make one step to the right, and go to state $q_1$.

   Read $b$, leave it $b$, make one step to the left, and go to state $q_f$.

   (d) $\# q_0 a b b \# a q_0 b b \# a b q_1 b \# a q_1 b b \# q_f b b \# q_f b \# q_f$.

4. Each instance of the PCP over a general $\Sigma$ can be encoded into an instance of the PCP over $\{0, 1\}$ as follows: replace each element of $\Sigma$ by $01^k0$ where $k$ is unique for each element. It is not hard to see that the original instance of the PCP over a general $\Sigma$ has a solution if and only if the encoded instance of the PCP over $\{0, 1\}$ has a solution. So decidability of the PCP over $\{0, 1\}$ would imply decidability of the PCP over general $\Sigma$. 

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5. Let $\Sigma = \{a\}$. Then we only need to consider the length of every string. Suppose the PCP instance consists of strings $w_1w_2\ldots w_n$ and $v_1v_2\ldots v_n$.

If there is a $i$ such that $|w_i| = |v_i|$, then this constitutes a solution.

If there is a $i$ with $|w_i| < |v_i|$ and a $j$ with $|w_j| > |v_j|$, then it is easy to see that there is a solution of the form

$$w_i \ldots w_iw_j \ldots w_j = v_i \ldots v_iv_j \ldots v_j$$

If $|w_i| > |v_i|$ for all $i$, or $|w_i| < |v_i|$ for all $i$, then there is no solution.