Datastructures and Algorithms

Assignment 2

Deadline: December 6, at 1.45pm

This deadline is hard as nails! Late submissions will not receive a grade.

Preferably hand in your solutions at the lecture; else send them to w.j.fokkink@vu.nl (in a single file).

1. Specify (or draw) the instructions of a nondeterministic TM that accepts exactly the infinite language of nonempty strings that consist of twice the same substring:

\[ \{ww \mid w \in \{a, b\}^+\}. \]

(For example, \(aa\), \(abab\) and \(baabaa\) are in this language, while \(ab\) and \(baaaab\) are not.)

Also explain the idea behind your TM. (Hint: Let the TM nondeterministically guess where the “second” substring \(w\) starts.) (20 pts)

2. Prove that a language \(L\) is recursive if and only if \(L\{\lambda\}\) is accepted by a TM that reaches a halt state on each (nonempty) input string.

(Your proof needs to argue both directions of this question.) (15 pts)

3. Let

\[
\begin{align*}
v_1 &= ab \\
v_2 &= aba \\
v_3 &= aba \\
v_4 &= b.
\end{align*}
\]

\[
\begin{align*}
w_1 &= a \\
w_2 &= ba \\
w_3 &= b \\
w_4 &= bba
\end{align*}
\]

(a) Is there a solution for the PCP? (1 pt)

(b) Is there a solution for the MPCP? (1 pt)

(c) How is this instance of MPCP reduced to an instance of PCP? (Such that the MPCP instance has a solution if and only if the resulting PCP instance has a solution.) (8 pts)
4. A cycle in an undirected graph is called *Hamiltonian* if this path in the graph visits each node exactly once.

The Hamiltonian cycle problem for a given undirected graph is whether a Hamiltonian cycle exists in the graph.

Sketch an explicit (nondeterministic, polynomial) construction to show that the Hamiltonian cycle problem is in NP. (15 pts)

5. Let $\Sigma = \{a, b\}$. The TM $M$, with $F = \{q_f\}$, is defined by:

$$
\begin{align*}
\delta(q_0, a) &= (q_0, a, R) \\
\delta(q_0, b) &= (q_1, b, R) \\
\delta(q_1, a) &= (q_0, a, R) \\
\delta(q_1, b) &= (q_2, b, R) \\
\delta(q_2, a) &= (q_f, a, L)
\end{align*}
$$

(a) Describe the language $L(M)$. (3 pts)

(b) Transform the question whether string $abba$ is in $L(M)$ into an instance of the bounded tiling problem. (8 pts)

(c) Show, using the transition function of $M$, that $abba \in L(M)$. (1 pt)

(d) Transform this computation of $M$ on input string $abba$ into a solution for the corresponding instance of the bounded tiling problem. (8 pts)