Datastructures and Algorithms: Exam 2

December 17, 2013

You are allowed to use copies of the slides during the exam.

1. Let $\Sigma = \{a\}$. Specify (or draw) a TM that accepts all $a$-strings of odd length. (12 pts)

2. Let $\Sigma = \{a, b\}$. The TM $M$, with $F = \{q_f\}$, is defined by:

\[
\begin{align*}
\delta(q_0, a) &= (q_0, a, R) \\
\delta(q_0, b) &= (q_1, b, R) \\
\delta(q_1, a) &= (q_0, a, R) \\
\delta(q_1, b) &= (q_f, b, L)
\end{align*}
\]

(a) Transform the question whether string $abb$ is in $L(M)$ into an instance of the MPCP. (7 pts)

(b) Show, by applying the transition function of $M$ to the tape, that $abb \in L(M)$. (1 pts)

(c) Transform this computation of $M$ on input string $abb$ into a solution for the corresponding instance of the MPCP. (15 pts)

3. Argue that NP is closed under union. (That is, if $L_1 \in \text{NP}$ and $L_2 \in \text{NP}$, then $L_1 \cup L_2 \in \text{NP}$.) (15 pts)
4. Let $\Sigma = \{a, b\}$. The nondeterministic TM $M$, with $F = \{q_f\}$, is defined by:

\[
\begin{align*}
\delta(q_0, a) &= \{(q_0, a, R)\} \\
\delta(q_0, b) &= \{(q_0, b, R), (q_1, b, R)\} \\
\delta(q_1, b) &= \{(q_f, b, L)\}
\end{align*}
\]

Transform the question whether string $aab$ is in $L(M)$ into an instance of the bounded tiling problem.

Moreover, show that this instance has no solution. (15 pts)

5. Let $f : \{0, 1\}^3 \to \{0, 1\}^3$ be defined as follows:

\[
\begin{align*}
f(000) &= f(0) = f(101) = 000 \\
f(001) &= f(01) = f(100) = 001 \\
f(010) &= f(11) = f(111) = 010 \\
f(011) &= f(110) = 011
\end{align*}
\]

Apply Simon’s algorithm to determine a (non-trivial) linear dependency between the digits of $s = 101$ (give one possible scenario). (25 pts)