1. Suppose that some NP-complete language $L_1$ is polynomial-time reducible to $L_2 \in \text{NP}$. Argue that $L_2$ is also NP-complete. (15 pts)

2. Let $\Sigma = \{a, b\}$. The TM $M$, with $F = \{q_f\}$, is defined by:

\[
\begin{align*}
\delta(q_0, a) &= (q_0, a, R) \\
\delta(q_0, b) &= (q_1, b, R) \\
\delta(q_1, a) &= (q_0, a, R) \\
\delta(q_1, b) &= (q_f, b, L)
\end{align*}
\]

(a) Transform the question whether string $abb$ is in $L(M)$ into an instance of the bounded tiling problem. (12 pts)

(b) Show, by applying the transition function of $M$ to the tape, that $abb \in L(M)$. (1 pt)

(c) Transform this computation of $M$ on input string $abb$ into a solution for the corresponding instance of the bounded tiling problem. (12 pts)

3. Transform the instance of the bounded tiling problem from the previous exercise into an instance of the satisfiability problem. (15 pts)
4. Let $f : \{0, 1\}^4 \to \{0, 1\}^4$ be defined as follows:

\[
\begin{align*}
  f(0000) &= f(1101) = 0100 \\
  f(0001) &= f(1100) = 0011 \\
  f(0010) &= f(1111) = 0010 \\
  f(0100) &= f(1001) = 0111 \\
  f(1000) &= f(0101) = 0101 \\
  f(0011) &= f(1110) = 1100 \\
  f(0110) &= f(1011) = 1011 \\
  f(1010) &= f(0111) = 1111
\end{align*}
\]

Apply Simon’s algorithm to determine a (non-trivial) linear dependency between the digits of $s = 1101$. (Give one possible scenario.) (25 pts)