Exercise Sheet 1

1. Sometimes an algorithm doesn’t always need the entire input to produce the correct output. Give an example of such an algorithm.

2. Why does the statement “the running time of algorithm $A$ is at least in $O(n^2)$” provide no information?

3. Is an algorithm with a worst-case time complexity in $O(n)$ always faster than an algorithm of which the worst-case time complexity is not in $O(n)$?

4. For each of the following functions $f$ and $g$, say whether $f = O(g)$ and/or $g = O(f)$.
   
   (a) $f(n) = 5n^2 + 3n + 7$ and $g(n) = n^3$.
   (b) $f(n) = \sum_{i=1}^{n} i$ and $g(n) = n^2$.
   (c) $f(n) = n^n$ and $g(n) = n!$.
   (d) $f(n) = n + n \log_2 n$ and $g(n) = n \sqrt{n}$.

5. Given the following algorithm, with $A[1..n]$ an array with $n$ integer values:

   ```
   maxSubArray(A[1..n])
   max ← 0
   for left ← 1 to n do
     sum ← 0
     for right ← left to n do
       sum ← sum + A[right]
       if sum > max then
         max ← sum
   return max
   ```

   What does this algorithm compute?
   What is the worst-case time complexity this algorithm (in terms of $O$ or $\Theta$)?
6. Consider the following definition of the power function, for $n \geq 0$:

$$p(x, n) = \begin{cases} 
1 & \text{if } n = 0 \\
x \cdot p(x, n - 1) & \text{if } n > 0
\end{cases}$$

Give a pseudocode description of an algorithm $\text{Power}(x, n)$ to compute the power function according to this definition. Argue that the number of recursive calls is in $O(n)$.

7. Consider the following definition of the power function, for $n \geq 0$:

$$q(x, n) = \begin{cases} 
1 & \text{if } n = 0 \\
q(x, \frac{n}{2})^2 & \text{if } n > 0 \text{ is even} \\
x \cdot q(x, \frac{n-1}{2})^2 & \text{if } n > 0 \text{ is odd}
\end{cases}$$

Give a pseudocode description of an algorithm $\text{Qower}(x, n)$ to compute the power function according to this definition. Argue that the number of recursive calls is in $O(\log n)$.

8. Perform the following sequence of operations on a stack, which initially is empty. Say after each operation what is the content of the stack.

$$\text{push}(5) \text{ push}(3) \text{ pop()} \text{ push}(2) \text{ push}(8) \text{ pop()} \text{ pop()} \text{ push}(9) \text{ push}(1)$$

9. How can a stack be employed to check whether the brackets in an arithmetic expression are placed correctly?

10. Explain in detail how two stacks can be implemented using one array of size $n$.
    Only if all places in the array are occupied, an overflow message may be given in response to a $\text{push}$ operation (to any of the two stacks).