1. Argue that the problem of finding a minimum spanning tree in an undirected weighted graph is in P.

2. Sketch an explicit (nondeterministic, polynomial) construction to show that the bounded tiling problem is in NP.

3. Let $\Sigma = \{a, b\}$. The TM $M$, with $F = \{q_f\}$, is defined by:
   \[
   \begin{align*}
   \delta(q_0, a) &= (q_0, a, R) \\
   \delta(q_0, b) &= (q_1, b, R) \\
   \delta(q_1, a) &= (q_0, a, R) \\
   \delta(q_1, b) &= (q_f, b, L)
   \end{align*}
   \]
   (a) Transform the question whether string $abb$ is in $L(M)$ into an instance of the bounded tiling problem.
   (b) Show, by applying the transition function of $M$ to the tape, that $abb \in L(M)$.
   (c) Transform this computation of $M$ on input string $abb$ into a solution for the corresponding instance of the bounded tiling problem.

4. Consider the following two adaptations of the definition of tiles for instructions $(r, b, R)$ and $(r, b, L)$ in $\delta(q, a)$, in the construction to show that the bounded tiling problem is NP-complete. In both cases explain what goes wrong.
   (a) The $r$ in the color at the sides of the tiles are omitted:
   
   \[
   \begin{array}{ccc}
   \square & R & R \square \\
   q, a & r, c & c \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \square & L & L \square \\
   r, c & c & q, a \\
   \end{array}
   \]
   \[
   (r, b, R) \in \delta(q, a) \\
   (r, b, L) \in \delta(q, a)
   \]

   (b) The $R$ in the color at the sides of the tiles for instructions $(r, b, R)$ and the $L$ in the color at the sides of the tiles for instructions $(r, b, L)$ are omitted:
   
   \[
   \begin{array}{ccc}
   \square & r & r \square \\
   q, a & r, c & c \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \square & r & r \square \\
   r, c & c & q, a \\
   \end{array}
   \]
   \[
   (r, b, R) \in \delta(q, a) \\
   (r, b, L) \in \delta(q, a)
   \]
5. Argue that P is closed under complement, union and intersection.

6. Argue that NP is closed under union and intersection.
   Also explain why the proof that P is closed under complement does not carry over to NP.

7. Suppose that some NP-complete language $L_1$ is polynomial-time reducible to $L_2 \in NP$.
   Argue that $L_2$ is also NP-complete.

8. Is it possibly impossible to determine whether P = NP holds?