Exercise Sheet 7

1. What is more general, an algorithm for directed or for undirected graphs?

2. Apply Dijkstra’s shortest path algorithm to the following undirected graph, with start node A.

   \[\text{You don’t need to make the underlying heap explicit.}\]

3. Suppose Dijkstra’s shortest path algorithm would terminate when the heap \(H\) contains one node (instead of when it has become empty).

   Would the algorithm then still be correct?

4. Given a directed weighted graph that consists of a cycle of three nodes and three edges with weight \(-1\). Show how the Bellman-Ford algorithm detects this cycle.
5. Compute a minimum spanning tree in the following undirected weighted graph:

(a) Apply Prim’s algorithm, starting in node \( s \).
(b) Apply Kruskal’s algorithm.

In both cases, give one possible scenario. (You don’t need to make the underlying heap or disjoint-set data structure explicit.)

6. Argue that the worst-case time complexity of Prim’s algorithm, implemented using a heap, on a weighted graph with \( n \) nodes and \( m \) edges, is \( O(m \cdot \log n) \).

7. Explain why it is crucial for the worst-case time complexity of Kruskal’s algorithm that in the disjoint-set data structure, when two disjoint sets are joined by the union-find algorithm, the larger set subsumes the smaller set.

What would be the worst-case time complexity of Kruskal’s algorithm if the smaller set could subsume the larger set?

8. Argue that if all edges in an undirected weighted graph carry a different weight, then the graph has a unique minimum spanning tree.

Give an example to show that if some edges in an undirected weighted graph carry the same weight, then the graph may have multiple different minimum spanning trees.