Exercise Sheet 9

1. In the proof that the halting problem is undecidable, why would it be wrong to first run \( H \) on \((M, w)\), and:
   - if this leads to a final state, conclude that \( w \notin L(M) \);
   - if this leads to a non-final halt state, run \( M \) on \( w \).

2. Let
   \[
   w_1 = 001 \quad w_2 = 0011 \quad w_3 = 11 \quad w_4 = 101 \\
   v_1 = 01 \quad v_2 = 111 \quad v_3 = 111 \quad v_4 = 010.
   \]
   (a) Is there a solution for the PCP?
   (b) Is there a solution for the MPCP?
   (c) How is this instance of MPCP reduced to an instance of PCP? (Such that the MPCP instance has a solution if and only if the resulting PCP instance has a solution.)

3. Let \( \Sigma = \{a, b\} \). The TM \( M \), with \( F = \{q_f\} \), is defined by:
   \[
   \delta(q_0, a) = (q_0, a, R) \\
   \delta(q_0, b) = (q_1, b, R) \\
   \delta(q_1, a) = (q_0, a, R) \\
   \delta(q_1, b) = (q_f, b, L)
   \]
   (a) Describe the language \( L(M) \).
   (b) Transform the question whether string \( abb \) is in \( L(M) \) into an instance of the MPCP.
   (c) Show, by applying the transition function of \( M \) to the tape, that \( abb \in L(M) \).
   (d) Transform this computation of \( M \) on input string \( abb \) into a solution for the corresponding instance of the MPCP.

4. Argue that if \( \Sigma \) consists of only two elements, then the PCP is still undecidable.

5. Argue that if \( \Sigma \) consists of a single element, then the PCP is decidable.