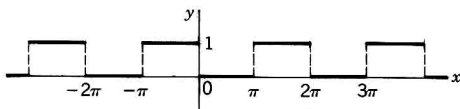


In each of the following problems you are given a function on the interval  $-\pi < x < \pi$ . Sketch several periods of the corresponding periodic function of period  $2\pi$ . Expand the periodic function in a sine-cosine Fourier series.

$$1. f(x) = \begin{cases} 1, & -\pi < x < 0, \\ 0, & 0 < x < \pi. \end{cases}$$

In this case the sketch is:



Your answer for the series is:  $f(x) = \frac{1}{2} - \frac{2}{\pi} \left( \frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right)$ .

$$2. f(x) = \begin{cases} 0, & -\pi < x < 0, \\ 1, & 0 < x < \frac{\pi}{2}, \\ 0, & \frac{\pi}{2} < x < \pi. \end{cases}$$

Answer:  $f(x) = \frac{1}{4} + \frac{1}{\pi} \left( \frac{\cos x}{1} - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} \dots \right) + \frac{1}{\pi} \left( \frac{\sin x}{1} + \frac{2 \sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} \dots \right)$ .

$$3. f(x) = \begin{cases} 0, & -\pi < x < \frac{\pi}{2}, \\ 1, & \frac{\pi}{2} < x < \pi. \end{cases}$$