Answer: $f(x) = \frac{1}{4} - \frac{1}{\pi}\left(\frac{\cos x}{1} - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} \cdots\right)$

$+ \frac{1}{\pi}\left(\frac{\sin x}{1} - \frac{2 \sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} - \frac{2 \sin 6x}{6} \cdots\right).$

4. $f(x) = \begin{cases} -1, & -\pi < x < \frac{\pi}{2}, \\ 1, & \frac{\pi}{2} < x < \pi. \end{cases}$

5. $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ -1, & 0 < x < \frac{\pi}{2}, \\ 1, & \frac{\pi}{2} < x < \pi. \end{cases}$

6. $f(x) = \begin{cases} 1, & -\pi < x < -\frac{\pi}{2}, \quad \text{and} \quad 0 < x < \frac{\pi}{2}; \\ 0, & -\frac{\pi}{2} < x < 0, \quad \text{and} \quad \frac{\pi}{2} < x < \pi. \end{cases}$

7. $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x, & 0 < x < \pi. \end{cases}$

Answer: $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right)$

$+ \left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right).$

8. $f(x) = 1 + x, \quad -\pi < x < \pi.$

Answer: $f(x) = 1 + 2(\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x \cdots).$

9. $f(x) = \begin{cases} x + \pi, & -\pi < x < 0, \\ 0, & 0 < x < \pi. \end{cases}$

Answer: $f(x) = \frac{\pi}{4} + \frac{2}{\pi}(\cos x + \frac{1}{9} \cos 3x + \frac{1}{25} \cos 5x \cdots)$

$- (\sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x \cdots).$

10. $f(x) = \begin{cases} x + \pi, & -\pi < x < 0, \\ -x, & 0 < x < \pi. \end{cases}$

11. $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ \sin x, & 0 < x < \pi. \end{cases}$

Answer: $f(x) = \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi}\left(\frac{\cos 2x}{2^2 - 1} + \frac{\cos 4x}{4^2 - 1} + \frac{\cos 6x}{6^2 - 1} \cdots\right).$