Resources are allocated to processors.
**Jobs**

A job is a unit of work, scheduled and executed by the system.

Parameters of jobs are:

- functional behavior
- time constraints
- resource requirements

Jobs are divided over processors, and they are competing for resources.

A scheduler decides in which order jobs are performed on a processor, and which resources they can claim.
**Terminology**

**release time**: when a job becomes available for execution

**execution time**: amount of processor time needed to perform the job (assuming it executes alone and all resources are available)

**response time**: length of time from arrival until completion of a job

**(absolute) deadline**: when a job is required to be completed

**relative deadline**: maximum allowed response time

**hard deadline**: late completion not allowed

**soft deadline**: late completion allowed

**jitter**: imprecise release and/or execution time

A preemptive job can be suspended at any time of its execution
Out of scope:

- use of distant resources
- communication between jobs
- migration of jobs
- overrun management
- penalty for missing a soft deadline
- performance
- different processor and resource types
Types of Tasks

A task is a set of related jobs.

A processor distinguishes three types of tasks:

- **periodic**: known input before the start of the system, and hard deadlines.
  Execution and interarrival times are fixed.

- **aperiodic**: executed in response to some external event, with soft deadlines.
  Execution and interarrival times are according to some probability distribution.

- **sporadic**: executed in response to some external event, with hard deadlines.
  Execution times are according to some probability distribution, and interarrival times are random.
**Periodic Tasks**

A periodic task is defined by:

- **release time** $r$ (of the first periodic job)
- **period** $p$ (regular time interval, at the start of which a periodic job is released)
- **execution time** $e$

For simplicity we assume that the relative deadline of each periodic job is equal to its period.

**Example:** $T_1 = (1, 2, 1)$ and $T_2 = (0, 3, 1)$.

The conflict at time 3 is resolved by some scheduler.

The **hyperperiod** is 6.
We focus on individual aperiodic and sporadic jobs.

- *Sporadic jobs* are only accepted when they can be completed in time.
- *Aperiodic jobs* are always accepted, and performed such that periodic and accepted sporadic jobs do not miss their deadlines.
**Average Response Time**

The queueing discipline of aperiodic jobs tries to minimize e.g. average *tardiness* (completion time minus deadline) or the number of missed soft deadlines.

The *average response time* of aperiodic jobs can be analyzed using:

- simulation and measurement
- Queueing Theory
- Integer Linear Programming

In the last two cases, values for e.g. average execution time of aperiodic jobs must in general still be estimated using simulation and measurement.
Scheduler

The scheduler of a processor schedules and allocates resources (according to some scheduling algorithms and resource access protocols).

A schedule is valid if:

- jobs are not scheduled before their release times
- the total amount of processor time assigned to a job equals its (maximum) execution time

A (valid) schedule is feasible if all hard deadlines are met.

A scheduler is optimal if it produces a feasible schedule whenever possible.
Clock-Driven Scheduler

Off-line scheduling: the schedule for periodic tasks is computed beforehand (typically with an algorithm for an NP-complete problem).

Time is divided into regular time intervals called frames.

In each frame, a predetermined set of periodic tasks is executed.

Jobs may be sliced into subjobs, to accommodate frame length.

Clock-driven scheduling is conceptually simple, but cannot cope with:

- jitter
- system modifications
- nondeterminism
**Slack**

Idle time in a frame can be used to execute aperiodic and sporadic jobs.

**Slack** of a frame \([s, c]\) at time \(t\) is \(t - c\) minus the total execution of periodic and accepted sporadic jobs in the frame after \(t\).

**Slack stealing**: execution of aperiodic jobs until the slack of the current frame is zero.

**Acceptance test**: straightforward check (in absence of jitter) whether a newly arrived sporadic job can be completed before its deadline (i.e., whether there is sufficient slack).

**Resources**: are in general distributed according to some precomputed cyclic schedule.
Example: Periodic jobs $T_1 = (0, 2, 1)$ and $T_2 = (0, 3, 1)$.

Frame length is 6.

Aperiodic job $A$, with execution time 2, arrives at 1.

Sporadic job $S$, with execution time 1, arrives at 2 with deadline 6.

Sporadic job $S'$, with execution time 1, arrives at 6 with deadline 7.
Priority-Driven Scheduling

On-line scheduling: the schedule is computed at run-time.

Scheduling decisions are taken when:

- periodic jobs are released or aperiodic/sporadic jobs arrive
- jobs are completed
- resources are required or released

Released jobs are placed in priority queues, e.g. ordered by:

- release time (FIFO, LIFO)
- execution time (SETF, LETF)
- period of the task (RM)
- deadline (EDF) or slack (LST)

We focus on EDF scheduling.
RM Scheduler

Rate Monotonic: Shorter period gives a higher priority.

Advantage: Priority on the level of tasks makes RM easier to analyze than EDF/LST.

Non-optimality of the RM scheduler (one processor, preemptive, no competition for resources):

Let $T_1 = (0, 4, 2)$ and $T_2 = (0, 6, 3)$.

Remark: If for periods $p < p'$, $p$ is always a divisor of $p'$, then the scheduler is optimal.
**EDF Scheduler**

**Earliest Deadline First**: the earlier the deadline, the higher the priority.

Consider a single processor, and preemptive jobs.

**Theorem**: When jobs do not compete for resources, the EDF scheduler is optimal.

Non-optimality in case of non-preemption:

```
  r_1  r_2  d_2  d_1
  ↓    ↓    ↓    ↓
  J_1  J_2  J_1  J_2
```

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>J_2</td>
<td></td>
<td>J_1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

EDF

non-EDF
Non-optimality in case of resource competition:

Let $J_1$ and $J_2$ both require resource $R$. 

\[ \begin{array}{cccc}
  & r_1 & r_2 & d_2 & d_1 \\
 0 & J_1 & J_2 & 1 & 2 & 3 & 4 \\
\end{array} \]
Non-optimality in case of two processors (with migration):

\[ r_1 \ r_2 \ r_3 \ d_1 \ d_2 \ d_3 \]

\[ J_1 \ J_2 \ J_3 \]

EDF

LST

Drawbacks of EDF:

- dynamic priority of periodic tasks makes it difficult to analyze which deadlines are met in case of overloads
- late jobs can cause other jobs to miss their deadlines (good overrun management is needed)
LST Scheduler

Least Slack Time first: less slack gives a higher priority.

Slack of a job at time $t$ is the idle time of the job until its deadline.

Theorem: When jobs do not compete for resources, the LST scheduler is optimal.

Remarks for the LST scheduler:

- Priorities of jobs change dynamically.
- Continuous scheduling decisions would lead to context switch overhead in case of two jobs with the same slack.
Non-optimality of the LST scheduler in case of two processors (with migration):

Drawback of LST: computationally expensive
Scheduling Anomaly

Let jobs be non-preemptive. Then shorter execution times can lead to violation of deadlines.

Consider the EDF (or LST) scheduler:

If jobs are preemptive, and there is no competition for resources, then there is no scheduling anomaly.
Utilization

Utilization of a periodic task \( T = (r, p, e) \) is \( \frac{e}{p} \).

Utilization of a processor is the sum of utilizations of its periodic tasks.

Assumptions: jobs preemptive, no resource competition.

Theorem: Utilization of a processor is \( \leq 1 \) if and only if scheduling its periodic tasks is feasible.

Example: \( T_1 = (1, 2, 1) \) and \( T_2 = (1, 2, 1) \).
Assignment of Periodic Tasks to Processors

Goal: To fit periodic tasks on a minimal number of processors.

Remark: Load balancing is not taken into account here.

Simple approach: Assume processors $P_1, \ldots, P_k$.

Periodic tasks $T_1, \ldots, T_\ell$ are assigned to processors as follows:

Let $T_1, \ldots, T_{i-1}$ have been assigned. $T_i$ is assigned to $P_j$ if it does not fit on $P_1, \ldots, P_{j-1}$ (i.e., utilization of these processors would grow beyond 1) and does fit on $P_j$.

Smart approach: First sort periodic tasks by their utilization ($T_1$ has largest utilization, $T_\ell$ smallest utilization).

This improves worst-case and average complexity (in number of required processors).
Example: Utilizations $\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3}$.

However, the smart approach is not optimal. Fitting periodic tasks on a minimal number of processors is NP-complete.

Example: Utilizations $\frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{2}{3}$.
Remark: Communication overhead between jobs on different processors, or between a job and a remote resource, may be taken into account.

The problem of assigning periodic tasks to processors with minimal communication overhead can be reformulated into Integer Linear Programming.
Scheduling Aperiodic Jobs

**Background**: aperiodic jobs are only scheduled in idle time.

**Drawback**: needless delay of aperiodic jobs.

**Slack stealing**: periodic tasks and accepted sporadic jobs may be interrupted if there is sufficient slack.

**Example**: \( T_1 = (0, 2, \frac{1}{2}) \) and \( T_2 = (1, 3, \frac{1}{2}) \).

Aperiodic jobs available in \([0, 6]\).

![Graph showing aperiodic jobs available between 0 and 6 with intervals at 1, 2, 3, 4, and 5 seconds.]

**Drawback**: difficult to compute in case of jitter.
Polling: gives a period $p_s$, and an execution time $e_s$ for aperiodic jobs in such a period.

At the start of a new period, the first $e_s$ time units can be used to execute aperiodic jobs.

Consider periodic tasks $T_k = (r_k, p_k, e_k)$ for $k = 1, \ldots, n$. The server works if
\[
\sum_{k=1}^{n} \frac{e_k}{p_k} + \frac{e_s}{p_s} \leq 1
\]

Drawback: aperiodic jobs released just after a polling may be delayed needlessly.

We proceed to present two servers based on polling that try to resolve this drawback.

For the moment, we ignore sporadic jobs.
Deferrable Server

Allows a polling server to save its execution time within a period (but not after this period!) if the aperiodic queue is empty.

In case of an EDF scheduler, the deadline of a deferrable server at the end of a period \( p_s \) can be treated as a hard deadline.

Then the deferrable server works if

\[
\sum_{k=1}^{n} \frac{e_k}{p_k} + \frac{e_s}{p_s} (1 + \frac{p_s - e_s}{p_i}) \leq 1
\]

for \( i = 1, \ldots, n \) (Ghazalie & Baker, 1995).
Remark: $\sum_{k=1}^{n} \frac{e_k}{p_k} + \frac{e_s}{p_s} \leq 1$ is not good enough.

Example: $T_1 = (2, 5, 3 + \epsilon)$ and $p_s = 3, e_s = 1$.

$T_1$ misses its deadline at 7

Drawback: only partial use of available bandwidth.
Total Bandwidth Server

Fix an allowed utilization rate $\tilde{u}_s$ for the server, such that

$$\sum_{k=1}^{n} \frac{e_k}{p_k} + \tilde{u}_s \leq 1$$

When the aperiodic queue is non-empty, a deadline $d$ is determined for the head of the queue, according to the following rules.

(Let the head of the aperiodic queue have execution time $e$.)

- When a job arrives at the empty aperiodic queue at time $t$:

  $$d := \max(d, t) + \frac{e}{\tilde{u}_s}$$

- When an aperiodic job completes and the tail of the aperiodic queue is non-empty:

  $$d := d + \frac{e}{\tilde{u}_s}$$

Initially, $d = 0$. 

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Aperiodic jobs can now be treated as periodic jobs by the EDF scheduler.

Example: $T_1 = (0, 2, 1)$ and $T_2 = (0, 3, 1)$. We fix $\bar{u}_s = \frac{1}{6}$.

$J$ released at 1 with $e = 2$ gets (at 1) deadline $1 + 12 = 13$.

$J'$ released at 2 with $e' = 1$ gets (at 12) deadline $13 + 6 = 19$.

**Drawback:** unfair in case of multiple servers; and computationally expensive compared to the deferrable server

**Generalized processor sharing** divides available time over servers in a round-robin fashion.
Acceptance Test for Sporadic Jobs

A sporadic job with deadline $d$ and execution time $e$ is accepted at time $t$ if utilization (of the periodic and accepted sporadic jobs) in the time interval $[t, d]$ is never more than $1 - \frac{e}{d-t}$.

If accepted, utilization in $[t, d]$ is increased with $\frac{e}{d-t}$.

Example: Periodic task $T_1 = (0, 2, 1)$.

Sporadic job with $r = 1$, $e = 2$ and $d = 6$ is accepted. Utilization in $[1, 6]$ is increased to $\frac{9}{10}$.

Sporadic job with $r = 2$, $e = 2$ and $d = 20$ is rejected.

Sporadic job with $r = 3$, $e = 1$ and $d = 13$ is accepted. Utilization in $[3, 6]$ is increased to 1, and utilization in $[6, 13]$ to $\frac{3}{5}$. 
The acceptance test may reject schedulable sporadic jobs.

**Example:** Periodic task $T_1 = (0, 2, 1)$.

A sporadic job is released at time 0 with $e = 1$ and $d = 1$. Utilization until time 1 is 1.5, but still the sporadic job could be scheduled.

**Remark:** The total bandwidth server can be integrated with an acceptance test for sporadic jobs (e.g. by making the allowed utilization rate $\tilde{u}_s$ dynamic.)
Resource units can be requested by jobs during their execution, and are allocated to jobs in a **mutually exclusive** fashion.

When a requested resource is refused, the job is preempted (blocked).

**Remark:** Resource sharing gives rise to **scheduling anomaly**.
**Dangers of Resource Sharing**

(1) Deadlock can occur.

**Example:** $J_1 > J_2$.

$J_2$ requires red resource, which yields deadlock.

$J_1$ requires green resource

(2) A job $J$ can be blocked by subsequent lower-priority jobs.

**Example:** $J > J_1 > \cdots > J_k$, and $J, J_k$ require the red resource.
Priority Inheritance

When a job $J$ requires a resource $R$ and becomes blocked, the job holding $R$ inherits the priority of $J$ until it releases $R$.

(1) Deadlock can still occur.

Example: $J_1 > J_2$.

(2) Blocking by subsequent lower-priority jobs becomes less likely.

Example: $J > J_1 > \cdots > J_k$, and $J, J_k$ require red.
Priority Ceiling

The priority ceiling of a resource $R$ at time $t$ is the highest priority of (known) jobs that require $R$ at some time $\geq t$.

The priority ceiling of the system at time $t$ is the highest priority ceiling of resources that are in use at time $t$.

(It has a special bottom value $\Omega$ when no resources are in use.)

In a priority ceiling protocol, from the arrival of a job, this job is not released until its priority is higher than the priority ceiling of the system.

Assumption: The resources required by a job are known beforehand.

Note: In the pictures to follow, $r$ denotes the arrival of a job.
(1) No deadlocks. Because a job can only start executing when all the resources it will require are free.

Example: \( J_1 > J_2 \).

(2) Blocking by subsequent lower-priority jobs becomes less likely.

Example: \( J > J_1 > \ldots > J_k \), and \( J, J_k \) require the red resource.

This example assumes that the arrival of \( J \) is known at time 1.

Question: What would happen if \( J \) were only known at its arrival?
**Preemption Ceiling**

**Motivation:** with dynamic priorities of tasks, the overhead of computing priority ceilings is high.

The **preemption level** of jobs must be such that if \( J > J' \), and \( J' \) arrives after \( J \), then \( J \) has a higher preemption level than \( J' \).

**Idea:** \( J' \) will never preempt \( J \).

**Example:** The preemption level can coincide with *priorities*, or with *arrival times*.

Ideally, the preemption level can be defined for periodic *tasks* (instead of jobs). For instance,

**EDF:** preemption level can coincide with RM-priorities of tasks

**FIFO:** any preemption level is allowed

**LIFO:** in general, preemption level cannot be defined for tasks
The preemption ceiling of a resource $R$ at time $t$ is the highest preemption level of (known) jobs that require $R$ at some time.

The preemption ceiling of the system at time $t$ is the highest preemption level of resources that are in use at time $t$.

In a preemption ceiling protocol, from the arrival of a job, the job is not released until its preemption level is higher than the preemption ceiling of the system.

(Moreover, there is a priority inheritance rule.)
(1) No deadlocks. Because a job can only start executing when all the resources it will require are free.

Example: \( J_1 > J_2 \).

(2) Blocking by subsequent lower-priority jobs becomes less likely.

Example: \( J > J_1 > \cdots > J_k \), and \( J, J_k \) require the red resource.

Note that the preemption level of \( J \) is the highest of all jobs.

This example assumes that the arrival of \( J \) is known at time 1.
Multiple Resource Units

The notions of *priority* and *preemption ceiling* assumed only one unit per resource type.

In case of multiple units of the same resource type, the definitions of priority and preemption ceiling need to be adapted:

The priority (or preemption) ceiling of a resource $R$ with $k$ free units at time $t$ is the highest priority (or preemption level) of known jobs that require $> k$ units of $R$ at some time $\geq t$.

Multiple Processors

In a multiprocessor setting, jobs that require a *global critical section* in general are given higher priority than jobs that only require local resources.
Greedy Synchronization Protocol

Consider a multiprocessor environment, where each processor runs some periodic tasks. There may be dependencies between jobs.

In the greedy synchronization protocol, jobs are released asap. This schedule is not always optimal!

**Example:** Two processors $P_1$ and $P_2$.

$P_1$ runs $T_1 = (0, 6, 3)$ and $T_2 = (0, 9, 3)$, with $T_1 > T_2$.

$P_2$ runs $T_3 = (0, 9, 3)$ and $T_4 = (6, 10, 5)$, with $T_3 > T_4$.

A job of $T_3$ can only be released if a job of $T_2$ completed.
**Example:** Consider the same example, but now with the EDF scheduler.

$P_1$ runs $T_1 = (0, 6, 3)$ and $T_2 = (0, 9, 3)$.

$P_2$ runs $T_3 = (0, 9, 3)$ and $T_4 = (6, 10, 5)$.

A job of $T_3$ can only be released if a job of $T_2$ completed.

All deadlines are met.

But the response time of a pair of jobs from $(T_2, T_3)$ can become long, and the completion time can become unpredictable.
Release-Guard Protocol

Suppose a job of periodic task \( T_2 = (r, p, e) \) can only be released if a job of periodic task \( T_1 \) completed.

In the release-guard protocol, the \( k \)-th job of \( T_2 \) is released at time \( t \) with:

- \( t \geq r + (k-1)p \);
- the \( k \)-th job of \( T_1 \) completed before \( t \); and
- either \( T_2 \)'s processor is idle at \( t \), or \( t \) is at least \( p \) time units after the release of the \( (k-1) \)-th job of \( T_2 \).
Example: Two processors $P_1$ and $P_2$.

$P_1$ runs $T_1 = (0, 3, 1)$ and $T_2 = (0, 4, 1)$.

$P_2$ runs $T_3 = (0, 4, 1)$ and $T_4 = (1, 3, 2)$.

A job of $T_3$ can only be released if a job of $T_2$ completed.

First consider the greedy synchronization protocol, with EDF.

Now consider the release-guard protocol, with $T_1 > T_2$ and $T_3 > T_4$.
Summary:

- clock-driven vs. priority-driven scheduling
- fixed priority of RM vs. dynamic priority of EDF/LST
- optimality of EDF/LST vs. non-optimality of RM
- divide periodic tasks over processors
- scheduling of aperiodic jobs
  (slack stealing / polling / total bandwidth)
- acceptance test for sporadic jobs (based on utilization)
- resource sharing
  (priority inheritance / priority ceiling / preemption ceiling)
- dependencies between jobs
  (greedy synchronization / release-guard)