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Abstract:

During the last three decades several different styles of *semantics* for programming languages have been developed. This thesis *compares* two of them: the *operational* and the *denotational* approach. We show how to give operational and denotational semantics to programming languages, and how to compare different semantic models for a given language. Both in the definition of the denotational semantics and in the comparison of semantic models we make use of *metric topology.* This thesis builts on the pioneering work by Arnold and Nivat and the foundational work by the Amsterdam Concurrency Group headed by De Bakker on the use of metric spaces in semantics.

The work reported in this thesis started with the search for *comparative semantics* for a fragment of ACPrρ, a real time language introduced by Baeten and Bergstra (1991). The principal construction of this language is the *integration* statement. In its most general form the integration statement gives rise to *unbounded nondeterminism.* We first considered a *finite* version of integration. An *operational semantics* was given for the language by means of a *labelled transition system.* Furthermore, a *denotational semantics* based on a *complete metric space* was constructed for the language. We proved that the two semantic models are equivalent by means of standard tools. Inspired by a theorem of Michael (1951), a result from general topology which roughly tells us that a compact union of compact sets is again compact, we next considered the language with a *compact* version of integration. The operational semantics could be extended without any problems. For the extension of the denotational semantics we needed some new ingredients. In the definition of the denotational semantics we exploited the already mentioned theorem of Michael and the fact that a continuous image of a compact set is compact, a result from general topology due to Alexandroff (1927). The equivalence proof of the two semantic models was the most difficult part to generalize. To prove the equivalence by *uniqueness of fixed point*, a proof principle based on Banach's unique fixed point theorem (1922) and introduced in semantics by Kok and Rutten (1990), the operational semantics should be *compact.* The compactness of an operational semantics is usually derived from the fact that the labelled transition system inducing the semantics satisfies the finiteness condition *finitely branching.* However, the labelled transition system at hand is not finitely branching. It does not even satisfy the weaker finiteness condition *image finite.* We provided the labelled transition system with some additional metric structure. The enriched labelled transition system was shown to be *compactly branching*, a generalization of finitely branching. This generalization enabled us to prove that the operational semantics is compact. The operational and denotational semantics and their equivalence proof are presented in Chapter 8.

In the above described comparative semantic study we used labelled transition systems enriched with metrics. We call these enriched labelled transition systems *metric labelled transition systems.* We continued our research by developing a general theory for metric labelled transition systems. Several unsuccessful attempts were made to generalize the other finiteness condition *image finite.* Restricting our attention to functions being nonexpansive, rather than continuous, enabled us to generalize image finite to *image compact.* We proved that an image compact metric labelled transition system induces a *closed* operational semantics, generalizing the folklore result that an image finite labelled transition system gives rise to a closed operational semantics. The theory of

metric labelled transition systems is presented in Chapter 7. This theory generalizes well known results of labelled transition systems. These results are presented in Chapter 4 and applied to give comparative semantics to a very simple language in Chapter 5.

Apart from the comparative semantic study described above we applied the theory of metric labelled transition systems to give comparative semantics to several other languages. Two further applications are presented in this thesis. An operational semantics and a denotational semantics for a language with *iteration* are related by uniqueness of fixed point in Chapter 9. A language with *second order communication* is modelled operationally and denotationally and the relationship of the two semantic models is studied in Chapter 10 (joint work with De Bakker).

In the study of metric labelled transition systems we encountered a new *branching domain*: a complete metric space of labelled trees. The elements of the branching domain, the *branching processes*, are endowed with a metric such that the distance of branching processes increases (exponentially) if the maximal depth at which the truncations of the branching processes coincide decreases (linearly). The branching domain was designed to model *image finite* language constructions, those constructions modelled operationally by means of an image finite labelled transition system. The *random assignment* is a standard example of an image finite language construction. The branching domain is used to give comparative semantics to a language with random assignment in Chapter 6. We were interested to see how this new branching domain relates to two already known branching domains introduced by De Bakker and Zucker (1982, 1983). The one domain was also introduced to model image finite language constructions, whereas the other domain was designed to model *finitely branching* constructions. The new domain turned out to be situated properly in between the two other domains. Although the new domain was designed to fit in between them, it took us some time before we could actually prove this. In the proof we employed various results from general topology including a theorem of Lindenbaum (1926), which had not been used before in semantics as far as we know. In the comparison of the branching domains we also used the fact that the branching domains are *compact* metric spaces. The study of the branching domains is presented in Chapter 3.

When the research reported in this thesis was started some problems of the branching domain for image finite constructions were already known. Bergstra and Klop (1987) had shown that the obvious *parallel composition* of two branching processes is in general not a branching process. Warmerdam (1990) had provided an intricate example showing that the same holds for the *sequential composition*. The new branching domain does not give rise to these problems. The sequential composition on the old and the new branching domain for image finite constructions is discussed in Section 6.3.

In the comparative study of the branching domains we exploited the fact that the branching domains are *compact* metric spaces. A rather ad hoc proof of this fact was first provided. In notes by Warmerdam (1991) we found some general theory, based on the theory of solving domain equations over *complete* metric spaces developed by America and Rutten (1989), also proving this fact. We worked out the details (joint work with Warmerdam). The main results can be found in Chapter 2.

Table of contents:

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