Motivate enyour answers. You may define and use a bisimulation without proving that it is a bisimulation (unless such a proof is explicitly asked).

Exercise 1. (6+6+4+4 points)
This exercise is concerned with truth and validity.

(a) Prove or disprove the universal validity of $\Diamond(p \land q) \rightarrow \Diamond p \land \Diamond q$.

(b) Prove or disprove the validity of $\Box \Diamond p \rightarrow \Diamond p$ in reflexive frames.

(c) Prove or disprove the following:
   for all $M, w$ and $\phi$: if $M, w \models \phi$ then $M, w \models \Box \phi$.

(d) Prove or disprove the following:
   for all $\phi$: if $\models \phi$ then $\models \Box \phi$.

Exercise 2. (3+6+6 points)
This exercise is concerned with bisimulations.

Consider the following model: $W = \{a, b, c\}$, and $R = \{(a, b), (a, c), (b, c), (c, a), (c, b)\}$, and $V(p) = \{b\}$.

(a) Show that the worlds $a$ and $b$ are not bisimilar.

(b) Prove or disprove: the worlds $a$ and $c$ are bisimilar.

(c) Give the bisimulation contraction of the model.

Exercise 3. (7+7 points)
This exercise is concerned with characterizations and definability.

(a) Asymmetry is defined as follows: $R_{xy} \rightarrow \neg R_{yx}$ for all $x, y$.
   Show that asymmetry is not definable in basic modal logic.

(b) The operator $A$ called global box is defined as follows:
   $M, w \models A \phi$ if and only if $M, v \models \phi$ for all worlds $v$.
   Show that the global box is not definable in basic modal logic.
Exercise 4. (7 points)
This exercise is concerned with decidability.

(a) Use a semantic tableau to find a counterexample against universal validity of $\Diamond p \land \Diamond q \rightarrow \Diamond (p \land q)$.

Exercise 5. (7+7+6 points) Consider the models $\mathcal{M}$ and $\mathcal{N}$ defined by:

(a) Show that there is no modal formula distinguishing state $n_3$ in model $\mathcal{N}$ from state $t$ in model $\mathcal{M}$.

(b) Let $\tilde{\mathcal{N}}$ be the PDL-extension of model $\mathcal{N}$. Compute the transition relation $\tilde{R}_\beta$ corresponding to the PDL-program $\beta = \text{while } p \text{ do } abba$.

(c) Determine whether the PDL-formula $[\beta]p \leftrightarrow p$ globally holds in $\tilde{\mathcal{N}}$. Prove your answer.
Exercise 6. \((4+5+5 \text{ points})\)

This exercise is concerned with epistemic logic with one agent.

We recall the definitions of the Hilbert systems:

- system \(K\) is the most basic system with the tautologies of first-order propositional logic as axioms, the axiom for modal distribution \((K(p \rightarrow q) \rightarrow (Kp \rightarrow Kq))\), the rule for modus ponens, the rule for necessitation (if \(\vdash \phi\) then \(\vdash K\phi\)), and the rule for substitution;

- system \(T\) is system \(K\) plus the truth axiom (or veridicality):
  \[ A1 : Kp \rightarrow p; \]

- system \(S4\) is system \(T\) plus the axiom of positive introspection:
  \[ A2 : Kp \rightarrow KKp; \]

- system \(S5\) is system \(S4\) plus the axiom of negative introspection:
  \[ A3 : \neg Kp \rightarrow K \neg Kp . \]

(a) Formulate the soundness and completeness theorem for system \(T\).

(b) Show that axiom \(A2\) is not derivable in system \(T\).

(c) Indicate and explain an error in the following derivation in \(T\):

1. \(q \rightarrow Kq\) (necessitation)
2. \(Kp \rightarrow KKp\) (substitution, 1)

The exam grade is (the total number of points plus 10) divided by 10. In addition, the bonus is added, and the maximum final grade is 10.