Exam Advanced Logic 2017-2018

Monday March 26, 2018, 12.00–14.45

6 exercises

Motivate your answers

Exercise 1. *(5+5+5+6 points)*
This exercise is concerned with basic modal logic (BML).

(a) Prove or disprove $\models \Diamond(p \to q) \to (\Box p \to \Diamond q)$.

(b) Prove or disprove $\models (\Box p \to \Box q) \to \Box(p \to q)$.

(c) Prove or disprove $\mathcal{F} \models \Box\Diamond\Box p \to \Diamond p$ for $\mathcal{F}$ a reflexive frame.

(d) Show that the formula $p \to \Box\Diamond p$ characterizes the symmetric frames (which is: $\forall xy\,(Rxy \to Ryx)$).

Exercise 2. *(4+5+5 points)*
This exercise is concerned with bisimulations.

Consider the frame $\mathcal{F}$ with set of states $W = \{0, 1\}$, and accessibility relation $R = \{(0, 0), (0, 1)\}$.

Consider also the frame $\mathcal{F}'$ with set of states $W' = \{a, b, c, d\}$, and accessibility relation $R' = \{(a, b), (a, c), (c, c), (c, d)\}$.

(a) Depict $\mathcal{F}$ and $\mathcal{F}'$.

(b) Extend both frames to models by using for both the empty valuation written here as W and W'.

Show that the pointed models $((\mathcal{F}, W), 0)$ and $((\mathcal{F}', W'), a)$ are bisimilar. (You do not have to prove that it is a bisimulation.)

(c) Consider now an unspecified valuation V for $\mathcal{F}$.

What valuation V' should we take for $\mathcal{F}'$ such that the pointed models $((\mathcal{F}, V), 0)$ and $((\mathcal{F}', V'), a)$ are bisimilar?

Exercise 3. *(6+6 points)*
This exercise is concerned with (non-)definablility.

(a) We define the operator E for 'universal diamond' as follows: $(W, R), V, x \models E\phi$ if and only if $\exists y \in W$ such that $(W, R), V, y \models \phi$.

Show that E is not definable in basic modal logic.

(b) Show that the frame property asymmetry (which is: $\forall xy$ if Rxy then $\neg Ryx$) is not modally definable.

Exercise 4. *(5+5 points)*
This exercise is concerned with decidability.
Use semantic tableaux or sequents to investigate the validity of the following propositions. If the proposition is not valid, give explicitly the countermodel that you found.

(a) $(\Diamond p \to \Diamond q) \to \Box(p \to q)$

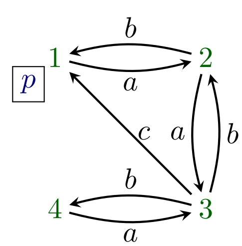
(b) $\Box(p \wedge q) \to (\Diamond p \vee \Box q)$

Exercise 5. *(6+6+6 points)*
This exercise is concerned with Propositional Dynamic Logic (PDL).

(a) Show that $\langle \alpha \cup \beta \rangle p \to \langle \alpha \rangle p \vee \langle \beta \rangle p$ is a PDL-tautology;

here α, β, γ are unspecified arbitry PDL-programs.

Now let $\widehat{\mathcal{M}}$ be the PDL-extension of the $\{a, b, c\}$-model $\mathcal{M}$ given by:



(b) Compute the relation $\hat{R}_\epsilon$ for the program $\delta = \textsf{while}\ \neg p\ \textsf{do}\ b \cup ac$.

Give also the intermediate results.

(c) Do we have $\mathcal{M} \models [\delta]p \to p$?

Exercise 6. *(5+5+5 points)*
This exercise is concerned with epistemic logic.
We recall the definitions of the Hilbert systems:

- system K is the most basic system with the tautologies of first-order propositional logic as axioms, the axiom for modal distribution ($K(p \to q) \to (Kp \to Kq)$), the rule for modus ponens, the rule for necessitation (if $\vdash \phi$ then $\vdash K\phi$), and the rule for substitution;
- system T is system K plus the truth axiom (or veridicality):
  $A1 : Kp \to p$;
- system $S4$ is system T plus the axiom of positive introspection:
  $A2 : Kp \to KKp$;
- system $S5$ is system $S4$ plus the axiom of negative introspection:
  $A3 : \neg Kp \to K\neg Kp$.

(a) Consider the following model:
$W = \{x, y, z, u\}$, $R = \{(x, y), (y, z), (z, x)\}$, $V(p) = \{y\}$.
Does this model show that $A2$ is not derivable in T?

(b) Show that $A3$ is not derivable in $S4$.

(c) Indicate and explain the error(s) in the following derivation in T:

1. $a \to b \to a$ (tautology)
2. $Kp \to p$ ($A1$)
3. $Kp \to p \to Kp$ (substitution, 1)
4. Kp (modus ponens, 3, 2)

The exam grade is (the total number of points plus 10) divided by 10.
In addition, the bonus is added, and the maximum final grade is 10.