VU University Amsterdam — exercise class 1

advanced logic 2018-2019 — Friday 2019 02 08

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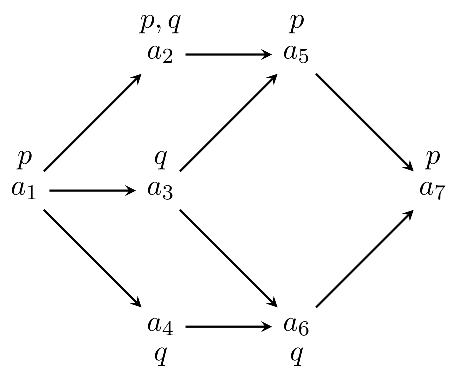
1. Consider the model $\mathcal{M} = (W, R, V)$ given by the following picture:




   (a) Write out the definitions of $W$, $R$ and $V$.

   (b) Show the following:

      (i) $\mathcal{M}, a_1 \models \Box\Box(p \vee q)$,

      (ii) $\mathcal{M}, a_2 \models \Diamond q \to \Diamond\Diamond q$,

      (iii) $\mathcal{M}, a_3 \models \Diamond p \to \Box(q \to \Box(p \to \Box p))$.

   (c) Show the following:

      (i) $\mathcal{M} \not\models p \to \Diamond p$,

      (ii) $\mathcal{M} \models \Box\Box\Box\neg q$,

      (iii) $\mathcal{M} \not\models q \to (\Diamond q \to \Box(q \to \Diamond q))$.

   (d) Change the valuation $V$ on the frame such that in the new model $\mathcal{M}' = (W, R, V')$ it holds that: $\mathcal{M}' \models \Box p \to p$.

2. The *binary tree* is the frame $\mathcal{B} = (W, R)$ where the domain $W$ is the set of all finite strings over the alphabet {0,1}:

$$W = \{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}\,.$$

Here $\varepsilon$ denotes the empty string, and the transition relation $R \subseteq W \times W$ is given by:

$$Rst \quad \text{if and only if} \quad t = s0 \text{ or } t = s1$$

   (a) Make a drawing of the first four levels of $\mathcal{B}$.

   (b) Consider a valuation $V$ on $\mathcal{B}$ that makes $p$ true on all strings of even length. Show that $\mathcal{B}, V \vDash \Box\Diamond p \to \Diamond\Box p$.

   (c) Let $V'$ be a valuation on $\mathcal{B}$ which makes the variable $p$ true on all strings whose first letter is 0, and $q$ on strings with first letter 1, so:

$$V'(p) = \{\, 0w \mid w \in \{0,1\}^* \,\}$$
$$V'(q) = \{\, 1w \mid w \in \{0,1\}^* \,\}$$

   Use this valuation to show that the formula $\lambda$ defined by:

$$\lambda = \Diamond p \wedge \Diamond q \to \Diamond(p \wedge \Diamond q) \vee \Diamond(p \wedge q) \vee \Diamond(\Diamond p \wedge q)$$

   is not valid in the binary tree.

   (d) Show that the formula $\Diamond\Diamond p \to \Diamond p$ is not valid in $\mathcal{B}$.

3. Prove or disprove universal validity of the following formulas:

   (a) $\Box(p \to q) \to (\Diamond p \to \Diamond q)$

   (b) $\Box(p \wedge q) \to (\Diamond p \wedge \Diamond q)$

   (c) $\Box p \to \Diamond p$

   (d) $\Diamond p \to \Box p$

   (e) $\Box(\Box p \to p) \to \Box p$