advanced logic
2018 03 22
lecture 14
overview

summary for the exam
validity: key notions

formulas of basic modal logic

\(((W, R), V), w) \models \phi \text{ formula } \phi \text{ is valid in a point } w

\(((W, R), V) \models \phi \text{ formula } \phi \text{ is valid in a model } ((W, R), V)

\( (W, R) \models \phi \text{ formula } \phi \text{ is valid in a frame } (W, R) \)

\models \phi \text{ formula } \phi \text{ is valid, or } \phi \text{ is a tautology}
satisfiability: key notions

formula $\phi$ is **satisfiable in model** $((W, R), V)$
if there is $w \in W$ such that $((W, R), V), w) \models \phi$

formula $\phi$ is **satisfiable**
if there is a model $((W, R), V)$ and $w \in W$ such that $((W, R), V), w) \models \phi$

validity and satisfiability are dual problems:

$\phi$ is valid if and only if $\neg \phi$ is not satisfiable
validity: things to know

if $\mathcal{F} \models \phi$ then $\mathcal{F} \models \phi^\sigma$

the modal tautologies

game semantics (mainly intuition)

alternative semantics (definition not asked)
validity: material

slides, in particular weeks 1–3

exercises

MLOM chapter 2
validity: typical exercises

prove or disprove validity of $\phi$ in a world or model or frame or globally

if a frame $\mathcal{F}$ has property $P$ then $\mathcal{F} \vDash \phi$
characterizations

characterize a world $w$ in a given model using a formula

\[ \phi \text{ is valid in } x \text{ if and only if } x = w \]

characterize a set of frames $\{\mathcal{F} | P(\mathcal{F})\}$ using a formula

\[ \phi \text{ is valid in } \mathcal{F} \text{ if and only if } \mathcal{F} \text{ has property } P \]

material: slides, exercises, MLOM chapter 2 and various other places
bisimulations and invariance

bisimulation and bisimilarity

bisimulation game

if two states a bisimilar then they are modally equivalent

if two states are modally equivalent in finitely branching models then they are modally equivalent

know the example motivating finitely branching
bisimulations and invariance: material

slides, in particular weeks 2–5

MLOM chapter 3
decidability

finite model property: result, no proof

translation to pred1 hardly discussed (so not in exam)

sequents

semantic tableaux

notation tableaux: annotate branching with "and" or with "or" to avoid confusion

slides, in particular week 6, exercises, MLOM chapter 4
temporal logic

basic modal logic with temporal (transitive and irreflexive) frames

basic temporal logic using \( \langle F \rangle \) and \( \langle P \rangle \)

basic temporal logic as instance of multi-modal logic with temporal model
temporal logic: important things

motivation model checking

until and next

operator is or is not definable

slides, in particular week 7, 8, MLOM chapter 7.4
multi-modal logic

zoom in on different accessibility relations

various instances of multi-modal logic

formulas, multi-modal frames, validity, bisimulation, invariance

slides week 9
propositional dynamic logic (PDL): key notions

motivation for PDL and connection with Hoare logic

formulas and programs of PDL

frames for PDL

model for PDL: extension of model for atomic programs

bisimilarity for atomic programs gives bisimilarity for regular programs

being safe for bisimulation
propositional dynamic logic: material

slides week 9-11, exercises, MLOM chapter 14.1–5

our starting point for validity is the definition on the slides
propositional dynamic logic: typical exercises

give the transition relation for some program

prove validity of a formula in a P DL-model or universally
epistemic logic: key notions

multi-modal logic with \([i]\) written as \(K_i\) for knowledge validity in a model and derivability in a proof system
soundness and completeness

modal distribution for all epistemic frames

add truth axiom or veridicality for reflexive frames

add postitive introspection for reflexive + transitive frames

add negative introspection for reflexive + transitive + symmetric frames
epistemic logic: material

slides 11-13

MLOM chapter 12 but not 12.4

MLOM chapter 8: also more generally about derivability
epistemic logic: typical exercises

model a simple setting
prove or disprove validity of a formula
you do not need to know the labels of the systems (A3, S4)
you need to know the descriptions (positive introspection)
being formal or clear of precise

give the type of a variable

give the quantifier and its scope

give the reason for a consequence

annotate a picture

motivate your answers

a claim that two states are bisimilar needs as justification a bisimulation

a relation can be claimed to be a bisimulation without proof
multi-modal logic
validity and derivability
bisimulation and invariance
soundness and completeness
decidability