advanced logic
2019 02 14
lecture 4

overview

- bisimulations

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motatívation bisimulation

what can be expressed by the modal language?

when can two pointed models $(\mathcal{M}, w)$ and $(\mathcal{M}', w')$ be distinguished?

when should two pointed models be considered modally identical?

what is the right semantic equivalence for the basic modal language?

recall: definition bisimulation between models

local harmony

zig

zag

recall: definition bisimularity

two models $\mathcal{M} = (W, R, V)$ and $\mathcal{M}' = (W', R', V')$ are bisimilar,

notation $\mathcal{M} \underline{\leftrightarrow} \mathcal{M}'$,

if there exists a bisimulation $Z \subseteq W \times W'$

two pointed models $\mathcal{M}, w$ and $\mathcal{M}', w'$ are bisimilar

notation $\mathcal{M}, w \underline{\leftrightarrow} \mathcal{M}', w'$ or $w \underline{\leftrightarrow} w'$,

if there exists a bisimulation $Z : \mathcal{M} \underline{\leftrightarrow} \mathcal{M}'$ such that $(w, w') \in Z$

property: bisimilarity between models is an equivalence relation

(reflexive, symmetric, and transitive)

remark: bisimulation versus two simulations

definition of a simulation: local harmony and only zig, no zag

question:

is $\mathcal{M}, w$ bisimilar with $\mathcal{N}, v$ the same as

$\mathcal{M}, w$ similar with $\mathcal{N}, v$ and also $\mathcal{N}, v$ similar with $\mathcal{M}, w$?

answer: no!

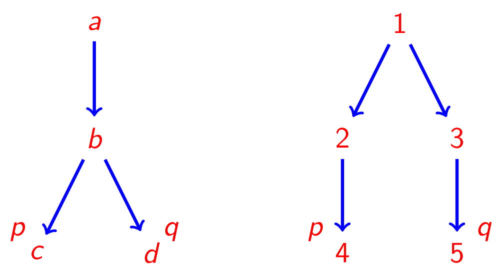
consider a in $(\{a, a_1, a'_1, a_2\}, \{(a, a_1), (a, a'_1), (a_1, a_2)\})$

and b in $(\{b, b_1, b_2\}, \{(b, b_1), (b_1, b_2)\})$

a simulates b and b simulates a but a and b are not bisimilar

think also of the formula $\Diamond\Box\bot$

example non-bisimilar states



the states a and 1 are not bisimilar

definition bisimulation games for two players

Spoiler S claims $\mathcal{M}, s$ and $\mathcal{N}, t$ to be different

Duplicator D claims they are similar

play consists of a sequence of links, starting with link $s \sim t$

at current link $m \sim n$ (with m in $\mathcal{M}$ and n in $\mathcal{N}$)

if m and n are different in their atoms then S wins

if not, then S picks a successor x either of m or of n

then D has to find a matching transition to y in the other model

play continues with next link $x \sim y$ (or $y \sim x$)

if a player cannot make a move, he loses; D wins the infinite games

definition modal equivalence

two states $\mathcal{M}, w$ and $\mathcal{M}', w'$ are modally equivalent

if they satisfy exactly the same formulas

theorem: bisimilarity implies modal equivalence

if $\mathcal{M}, w$ and $\mathcal{M}', w'$ are bisimilar then they are modally equivalent

so also: if two states are not modally equivalent, then they are not bisimilar

the proof is by induction on the definition of formulas

frame properties

for some properties P and formulas ϕ we can show

$\mathcal{F} \models \phi$ if and only if $P(\mathcal{F})$ for every frame $\mathcal{F}$

using bisimilarity implies modal equivalence

we can for some properties P show:

P is not modally definable

asymmetry (if Rxy then $\neg Ryx$) is not modally definable

suppose there is a formula ϕ characterizing asymmetry

we use the natural numbers frame $\mathcal{N}$ and the even–odd frame $\mathcal{F}$

we have $\mathcal{N} \models \phi$ but not $\mathcal{F} \models \phi$

let V be a valuation for $\mathcal{F}$ and let x be a state in $\mathcal{F}$

we aim to show $\mathcal{F}, V, x \models \phi$ (to derive a contradiction)

we take $x = e$ (other case similar)

we construct W such that $\mathcal{F}, V, e$ and $\mathcal{N}, W, 0$ are bisimilar

hence $\mathcal{F}, V, e$ and $\mathcal{N}, W, 0$ are modally equivalent

because $\mathcal{N} \models \phi$ we have $\mathcal{N}, W, 0 \models \phi$ and hence $\mathcal{F}, V, e \models \phi$

hence $\mathcal{F} \models \phi$ which yields a contradiction

hence there is no formula ϕ characterizing asymmetry