An Interpretation of Default Logic in Minimal Temporal Epistemic Logic

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Abstract

When reasoning about complex domains, where information available is usually only partial, nonmonotonic reasoning can be an important tool. One of the formalisms introduced in this area is Reiter's Default Logic (1980). A characteristic of this formalism is that the applicability of default (inference) rules can only be verified in the future of the reasoning process. We describe an interpretation of default logic in temporal epistemic logic which makes this characteristic explicit. It is shown that this interpretation yields a semantics for default logic based on temporal epistemic models. A comparison between the various semantics for default logic will show the differences and similarities of these approaches and ours.

Keywords

Nonmonotonic reasoning, default logic, temporal logic, epistemic logic, preferential entailment.

1 Introduction

Many complex reasoning tasks have to deal with incomplete information. Reasoning systems trying to accomplish such a task contain basic knowledge that can be used to draw some conclusions about the domain, but may also have to make reasonable assumptions to be able to draw further conclusions. These assumptions usually reflect common sense knowledge about the domain, but sometimes have to be retracted in view of new evidence. A characteristic of this type of reasoning is that often several (conflicting) sets of assumptions exist which all lead to a coherent description of the domain. Any formalization of such reasoning processes has to take this into account. One of the logical approaches is Reiter's default logic (e.g., see (Reiter 1980), (Besnard 89) and (Lukaszewicz 1990)). A characteristic of default logic is that only after a complete set of (default) assumptions has been chosen, it can be checked whether it indeed gives an acceptable description of the domain. To be more specific, during the reasoning one meets a specific type of conditions (called *justifications*) in default rules to be applied, that cannot be fulfilled only on the basis of what has been derived until that moment. After application of a default rule, only in the *future of the reasoning process* it can be verified whether such a condition of the applied default rule will turn out to be justified or to be defeated. Therefore there is an essential temporal element in default reasoning. This suggests that a default rule can be given an interpretation as a temporal rule with one of its conditions referring to the future of the reasoning process. It seems that the process of actually constructing a set of coherent assumptions is reflected in Reiter's approach to a certain extent, but without making the essential temporal element explicit. In this paper we will describe an interpretation of default logic in temporal epistemic logic. Notice that by temporal here a reference to *internal* time is meant: the reasoning is about a domain at a certain fixed point in time, but the reasoning system itself has an internal time over which its knowledge can vary.

The main contribution of this semantics for default logic when compared to other approaches is that the temporal element in default reasoning is made explicit in the semantics. This enables better insight in the inherent dynamic nature of default logic. In a sense, our temporal rules prescribe the agent to perform the action of applying a default rule. Viewed in this manner, our semantics have an analogous underlying idea as the approach of Lin and Reiter who give semantics for Logic Programming in the Situation Calculus (Lin and Reiter 1996).

During a reasoning process the truth or falsity of formulae may be unknown at some time, but when certain additional assumptions have been made, and new conclusions have been drawn, the truth (or falsity) of these formulae may have become known (derived) at a later time. The dynamics of such a reasoning process can be described by transitions of information states, or epistemic states, representing what has been found to be true at a certain time. Since the derived knowledge in a reasoning process will vary over time, we need epistemic states that may vary in time: *temporal epistemic models*. Earlier an approach to describe complex reasoning patterns (performed by meta-level architectures) using temporal semantics turned out to be successful (see (Treur 1994)), and this idea can be used to give semantics to other reasoning formalisms as well (see (Engelfriet and Treur 1994a), (Engelfriet, Herre and Treur 1995)).

In Section 2 the basic concepts of the (linear discrete time) temporal epistemic logic we need are introduced, with three modal operators referring to time. Then in Section 3 a short overview of Reiter's default logic is given. In Section 4 an interpretation mapping of default rules according to Reiter's approach into formulae of temporal epistemic logic is introduced. In Section 5 it is shown that this interpretation mapping preserves semantics: Reiter extensions are translated into (minimal) temporal epistemic models and vice versa, after which the main correspondence theorem of this paper is established. This correspondence theorem defines a one-to-one correspondence between default extensions of default theories in Reiter’s sense and a class of (minimal) linear discrete time temporal epistemic models. Based on this it is shown that under the interpretation mapping, sceptical entailment for default logic corresponds to minimal entailment for temporal epistemic logic. This interpretation yields a temporal semantics for default logic. In Section 6 a comparison is made to other approaches giving semantics to default logic. The proofs of our results can be found in an appendix.

An abstract of a preliminary version of this paper, which used partial models instead of epistemic states, appeared in (Engelfriet and Treur 1993).

2 Temporal epistemic logic

In this section we introduce a temporal epistemic logic to describe reasoning behaviour. Our approach is in line with what in (Finger and Gabbay 1992) is called temporalising a given logic; in our case the given logic is S5. For more literature on temporal logic, see for instance (Van Benthem 1983). As the base language in which the reasoning agent can express its knowledge and conclusions, we will take a propositional language:

Definition 2.1 (signature)

A *signature* $\mathbf{\Sigma}$ is an ordered sequence of (propositional) atom names.

The formulae of the language will be the classical propositional formulae. A state in a reasoning process should contain the information which has been derived at that point. We will formalise these states as follows:

Definition 2.2 (epistemic state)

a) A set of propositional models of signature $\mathbf{\Sigma}$ is called *closed* if there is a consistent set of formulae of which it is the model class.

An *epistemic state* (or shortly *state*), based on $\mathbf{\Sigma}$, is a closed set of propositional models of signature $\mathbf{\Sigma}$.

The set of epistemic states based on $\mathbf{\Sigma}$ is denoted by $\mathbf{S(\Sigma)}$, or shortly $\mathbf{S}$.

b) The truth of a propositional formula α in an epistemic state $\mathbf{M}$, denoted $\mathbf{M} \models \alpha$, is defined by:

$$\mathbf{M} \models \alpha \quad \Leftrightarrow \quad \mathbf{m} \models \alpha \text{ for all } \mathbf{m} \in \mathbf{M}$$

c) The state $\mathbf{N}$ is called a *refinement* of the state $\mathbf{M}$, denoted by $\mathbf{M} \leq \mathbf{N}$, if $\mathbf{M} \supseteq \mathbf{N}$.

d) The *theory* of a state $\mathbf{M}$, denoted $\mathbf{Th(M)}$ is defined by $\mathbf{Th(M)} =_{def} \{ \alpha \mid \mathbf{M} \models \alpha \}$.

e) For a set of formulae **S**, the set of models of **S** is denoted by **Mod(S)**.

f) For a set of formulae **S**, the deductive closure of **S** is denoted by **Cn(S)**.

Note that if $\mathbf{M \le N}$ then **N** contains more information than **M**: $\mathbf{Th(M) \subseteq Th(N)}$. The restriction to closed sets has been made to simplify definitions and proofs.

We will now temporalise these states, using linear discrete time with a starting point. For convenience we will take the set of natural numbers $\mathbb{N} = \{0, 1, 2, ...\}$ as the time frame.

Definition 2.3 (temporal epistemic model)

Let Σ be a signature.

a) A (propositional) *temporal epistemic model* **M** of signature Σ is a mapping

$$\mathbf{M}: \mathbb{N} \rightarrow \mathbf{S}(\Sigma)$$

We will sometimes use the notation $(\mathbf{M_t})_{t \in \mathbb{N}}$ where each $\mathbf{M_t}$ is an epistemic state as an equivalent description of a temporal epistemic model **M**.

A temporal epistemic model **M** is called *conservative* if for all time points **s** and **t** with $\mathbf{s \le t}$ it holds $\mathbf{M_s \le M_t}$.

b) The refinement relation $\le$ between temporal epistemic models is defined as: $\mathbf{M \le N}$ if for all time points **t** it holds $\mathbf{M_t \le N_t}$.

So a conservative temporal epistemic model can describe the knowledge over time of a reasoning agent which does not forget or revise its knowledge.

We can evaluate propositional formulae in epistemic states at any point in time, reflecting the knowledge of the reasoner at that moment, but we will need a language to talk about the change of knowledge over time. To this end we introduce a temporal epistemic language and give its semantics. We introduce two temporal operators, **F** and **G**, referring to future states. Intuitively, the temporal formula $\mathbf{F}\alpha$ is true at time **t** if viewed from time point **t**, the formula α will be known (to be true) at *some* time in the future (in *some* future state), and $\mathbf{G}\alpha$ is true at time t if α will be known (to be true) at *all* times in the future (in *all* future states). Furthermore we will need an operator that expresses the fact that *currently* α is known to be true (in the *current* state); this will be the operator $\mathbf{C}$. Although in this paper we will not need nested temporal operators it is easy to extend the definition:

Definition 2.5 (temporal epistemic language and semantics)

a) The temporal language $\mathcal{L}_T$ is the least set satisfying:

(i) if α is a propositional formula then $\mathbf{F}\alpha, \mathbf{G}\alpha, \mathbf{C}\alpha \in \mathcal{L}_T$

(ii) if $\alpha, \beta \in \mathcal{L}_T$ then $\neg\alpha, \alpha \wedge \beta \in \mathcal{L}_T$

b) The truth of a formula $\alpha \in \mathcal{L}_T$ in a temporal epistemic model $\mathbf{M}$ at time point $t \in \mathbb{N}$, denoted by $(\mathbf{M}, t) \models \alpha$ is defined inductively by

(i) $(\mathbf{M}, t) \models \mathbf{F}\alpha \Leftrightarrow$ there exists $s \in \mathbb{N}$ such that $s > t$ and $\mathbf{M}_s \models \alpha$

$(\mathbf{M}, t) \models \mathbf{G}\alpha \Leftrightarrow$ for all $s \in \mathbb{N}$ such that $s > t$ it holds $\mathbf{M}_s \models \alpha$

$(\mathbf{M}, t) \models \mathbf{C}\alpha \Leftrightarrow \mathbf{M}_t \models \alpha$

(ii) For $\alpha, \beta \in \mathcal{L}_T$:

$(\mathbf{M}, t) \models \neg\alpha \Leftrightarrow$ it is not the case that $(\mathbf{M}, t) \models \alpha$

$(\mathbf{M}, t) \models \alpha \wedge \beta \Leftrightarrow (\mathbf{M}, t) \models \alpha$ and $(\mathbf{M}, t) \models \beta$

c) For a temporal epistemic model $\mathbf{M}$ and $\alpha \in \mathcal{L}_T$, define $\mathbf{M} \models \alpha$ if $(\mathbf{M}, t) \models \alpha$ for all $t \in \mathbb{N}$, and for a set $\mathbf{K} \subseteq \mathcal{L}_T$ define $\mathbf{M} \models \mathbf{K}$ if $\mathbf{M} \models \alpha$ for all $\alpha \in \mathbf{K}$.

For a set of temporal epistemic models $\mathbb{M}$ and a propositional formula α, define $\mathbb{M} \models \alpha$ if $\mathbf{M} \models \alpha$ for all $\mathbf{M} \in \mathbb{M}$.

d) Equivalence of two formulae in temporal epistemic logic is defined as follows: $\alpha \equiv \beta$ if for all temporal epistemic models $\mathbf{M}$ and all $t \in \mathbb{N}$, $(\mathbf{M}, t) \models \alpha \Leftrightarrow (\mathbf{M}, t) \models \beta$. Now we define disjunction and implication by:

$$\alpha \vee \beta \equiv_{def} \neg(\neg\alpha \wedge \neg\beta)$$

$$\alpha \rightarrow \beta \equiv_{def} \neg\alpha \vee \beta$$

The $\mathbf{C}$ operator is very similar to the modal $\mathbf{K}$ operator, so for instance the formula $\neg\mathbf{C}\alpha \wedge \neg\mathbf{C}\neg\alpha$ denotes that α is unknown (i.e., neither true nor false).

In contrast with traditional temporal logics, it is not the case that $G\alpha \equiv \neg F \neg \alpha$. For example, take for α an atom p, and a temporal epistemic model M with for all t, M_t is the set of all models, then obviously $(M, 0) \not\models Gp$, whereas no $s \in \mathbb{N}$ exists with $M_s \models \neg p$ and thus $(M, 0) \not\models F\neg p$ so that $(M, 0) \models \neg F\neg p$.

The temporal models that are conservative can be axiomatised by the set C' consisting of the temporal axioms

$$C(\alpha) \rightarrow G(\alpha)$$

for every propositional formula α; this rule states that if α is known (to be true) at a certain point in time t, then α should also be known (to be true) in M at all time points in the future. It is easy to verify that these rules characterise the conservative temporal epistemic models:

Lemma 2.4

Let $C' = \{ C(\alpha) \rightarrow G(\alpha) \mid \alpha$ a propositional formula $\}$. For any temporal epistemic model M it holds: M is conservative if and only if $M \models C'$

The idea behind our approach of giving semantics to reasoning processes based on temporal epistemic logic (see (Engelfriet and Treur 1994a)) is that we use a temporal theory to describe the knowledge of a reasoning agent over time. This theory prescribes the nonmonotonic inferences which have to be performed. But this should also be *all* the agent knows (see for instance (Halpern and Moses 1984)) . To formalise this idea we will use minimality with respect to the ordering $\leq$:

Definition 2.6

(i) Let T be a temporal theory ($T \subseteq \mathcal{L}_T$). A temporal epistemic model M is called a *minimal (temporal) model of* T, denoted $M \models_{min} T$ if:

a) $M \models T$, and

b) For all temporal epistemic models $\mathbf{N}$ with $\mathbf{N} \leq \mathbf{M}$ and $\mathbf{N} \models \mathbf{T}$ it holds $\mathbf{N} = \mathbf{M}$.

The set of all minimal temporal models of $\mathbf{T}$ is denoted by $\mathbb{M}(\mathbf{T})$.

(ii) Based on minimal models we define the (sceptical) nonmonotonic consequence relation $\models_{\mathbf{min}}$ between theories and formulae by:

$$\mathbf{T} \models_{\mathbf{min}} \varphi \quad \Leftrightarrow \quad \mathbb{M}(\mathbf{T}) \models \varphi$$

Temporal epistemic models can be used to describe the sequence of epistemic states induced by a reasoning process. Its state will change over time, as a result of applying (nonmonotonic) inference rules to the set of what is currently known and possibly as a result of new (either observed or assumed) information about the domain. Usually one is interested in the final outcome of the reasoning process. If this process is modelled by a temporal epistemic model then this means that we want to know the truth of formulae as time goes to infinity. We assume that the reasoning is conservative: once a fact is known to be true or false (at a certain point in time $\mathbf{t}$), it will remain so at all later time points. Under this assumption, a propositional formulae will either remain unknown at all points in time (its truth or falsity is never established), or it is found to be true or false at a certain moment, and will remain so forever. Thus, the final outcome of the reasoning process can be viewed as the limit of $\mathbf{M_t}$ for time goes to infinity.

Definition 2.7 (limit of a conservative temporal epistemic model)

The *limit* of a conservative temporal epistemic model $\mathbf{M}$, denoted by $\mathbf{lim\,M}$, is defined by:

$$\mathbf{lim\ M} = \bigcap_{\mathbf{t=0}}^{\infty} \mathbf{M_t}$$

Using compactness of propositional logic, conservativity of $\mathbf{M}$ and the fact that for each $\mathbf{t}$, $\mathbf{M_t}$ is closed and non-empty the following can easily be checked:

Lemma 2.8

Let $\mathbf{M}$ be a conservative temporal epistemic model.

a) The limit $\mathbf{lim\,M}$ of $\mathbf{M}$ is an epistemic state (it is closed), and

$$\mathbf{Th}(\lim \mathbf{M}) = \bigcup_{t=0}^{\infty} \mathbf{Th}(\mathbf{M}_t).$$

b) For any propositional formula φ it holds

$$\lim \mathbf{M} \vDash \varphi \quad \Leftrightarrow \quad \mathbf{M} \vDash \mathbf{F}(\varphi)$$

For a detailed study of the nonmonotonic temporal epistemic logic described here, see (Engelfriet 1996).

3 Reiter's default logic

This section will present a brief overview of Reiter's default logic. Although Reiter's definitions were for any first-order language, we will restrict ourselves to propositional logic, as is commonly done. A *default rule* (or *default*) of signature Σ is an expression of the form $(\alpha : \beta_1, \ldots, \beta_n) / \gamma$ where $\alpha, \beta_1, \ldots, \beta_n$ and γ are propositional formulae of signature Σ. Intuitively such a default rule means: if α is believed and it is not inconsistent to assume β_1 through β_n, then assume γ. A *default theory* Δ is then a pair $< \mathbf{W}, \mathbf{D} >$ with $\mathbf{W}$ a set of sentences of signature Σ (the *axioms* of Δ) and $\mathbf{D}$ a set of default rules of signature Σ. In the rest of this paper we will assume that the signature Σ is fixed. For the definition of a Reiter extension of a default theory, we will not only give Reiter's definition ((Reiter 1980); also see (Besnard 1989), (Lukaszewicz 1990)), but also an equivalent one. The reason is that we want to obtain a correspondence between certain temporal epistemic models and Reiter extensions. It is easy to see that the two conditions as formulated in Lemma 3.1 are equivalent:

Lemma 3.1

Let $\Delta = < \mathbf{W}, \mathbf{D} >$ be a default theory of signature Σ, and let $\mathbf{E}$ be a set of sentences for Σ. Then the following conditions are equivalent:

(i) $\mathbf{E} = \bigcup_{i=0}^{\infty} \mathbf{E}_i$

where

$$E_0 = Cn(W),$$

and for all $i \geq 0$

$$E_{i+1} = Cn(E_i \cup \{ \omega \mid (\alpha : \beta_1, \ldots, \beta_n) / \omega \in D, \alpha \in E_i \text{ and }$$
$$\neg \beta_1 \notin E, \ldots, \neg \beta_n \notin E \})$$

(ii) $E = \bigcup_{i=0}^{\infty} E_i'$

where

$$E_0' = W,$$

and for all $i \geq 0$

$$E_{i+1}' = Cn(E_i') \cup \{ \omega \mid (\alpha : \beta_1, \ldots, \beta_n) / \omega \in D, \alpha \in Cn(E_i') \text{ and }$$
$$\neg \beta_1 \notin E, \ldots, \neg \beta_n \notin E \}$$

Definition 3.2 (Reiter extension)

A set of sentences E is called a *Reiter extension* of a default theory $\Delta = \langle W, D \rangle$, if (any of) the two conditions of Lemma 3.1 are satisfied. If E is a Reiter extension, then throughout the paper by E_i we will denote the subsets of E as defined by condition (i).

If E is a Reiter extension, then it is easy to see that for any $i \in \mathbb{N}$ it holds that $E_i \subseteq E_{i+1}$, $E_i = Cn(E_i)$ and that $E = Cn(E)$.

4 A temporal interpretation of default logic

The temporal view on reasoning processes introduced in (Engelfriet and Treur 1994a), (Treur 1994) can be applied to default reasoning as follows. Let $\Delta = \langle W, D \rangle$ be a default theory. A trace of a default reasoning process based on Δ is described by a sequence of epistemic states with increasing information. Such a trace is formalised by a conservative temporal epistemic model M; i.e., M is a temporal model of C'. The initial state of a default reasoning trace (and therefore all subsequent states as well) includes **W**; therefore the temporal model **M** should make **W** true: **M** is a model of $\{ C\alpha \mid \alpha \in W \}$.

If $(\alpha : \beta_1, ..., \beta_n) / \gamma$ is a default rule, then in order to simplify the notation we will sometimes take $n = 1$. As can be checked this makes no essential difference in any of the definitions and proofs further on. Suppose a default rule $(\alpha : \beta) / \gamma$ is given with propositional formulae α, β, γ and α is true in **M** at time point **t**; i.e., $(M, t) \models C\alpha$. If the default rule is applicable then its consequent is required to be true in the next state (and by conservativity in all subsequent states); i.e., $(M, t) \models G\gamma$. What remains is how to express whether application of the default rule is justified. The requirement is that β is consistent in the context of the reasoning process, including the part of the context yet to be generated by further reasoning steps. Since the reasoning is conservative, this means that there should be no future state where $\neg\beta$ is generated. In the temporal logic we designed this is quite easy to express: it is required that $(M, t) \models \neg F(\neg \beta)$. If we compare this to the translation of the justification for nonmonotonic modal logics as defined in (Marek, Schwarz and Truszczynski 1993), $LM\beta$ (the agent should know that it considers β possible), one sees that our translation is the dynamic variant of their translation. The agent considers β possible just in case she never derives $\neg\beta$. Summarising, for our temporal model we require: if α is currently known to be true at time point **t** and $\neg\beta$ is not known to be true at any time point after **t**, then γ should be known to be true at all time points after **t**. In temporal epistemic logic, this translates into the formula:

$$C\alpha \ \wedge \ \neg F(\neg \beta) \ \rightarrow \ G\gamma$$

This leads us to the following definition:

Definition 4.1 (temporal interpretation mapping for default theories)

Let $\Delta = < W, D >$ be a default theory.

a) The mapping τ , associating with any default rule $(\alpha : \beta_1, ..., \beta_n) / \gamma$ its *temporal interpretation*, is defined by

$$\tau : (\alpha : \beta_1, ..., \beta_n) / \gamma \quad \mapsto \quad C\alpha \wedge \neg F(\neg \beta_1) \wedge \ldots \wedge \neg F(\neg \beta_n) \rightarrow G\gamma$$

The set $\tau(D)$ is called the *temporal interpretation* of the set of default rules D.

b) The *temporal interpretation* of W is defined by $\tau(W) =_{def} \{ C\alpha \mid \alpha \in W \}$.

c) The *temporal interpretation of the default theory* Δ is defined by:

$$\tau(\Delta) = \tau(W) \cup \tau(D) \cup C'$$

The temporal interpretation of Δ ensures that any default rule which is applicable, is actually applied. However, we also want to make sure that these default conclusions are *the only ones* which are added to the knowledge of the reasoner. As will be seen in the next section, this can be accomplished by taking the minimal temporal models of the interpretation of the default theory.

As an aside, the set C' will turn out to be superfluous. We retain it in the definition since it simplifies some of the proofs.

After the introduction of the interpretation of default theories in temporal theories in the current section, in Section 5 we will establish what the precise requirements are in order to obtain a semantical one-to-one correspondence: between Reiter extensions of the default theory and minimal temporal models of its temporal interpretation.

5 Semantical correspondences underlying the interpretation

In the previous section we defined a correspondence between (sets of) default rules and (sets of) temporal formulae at a syntactic level, and we gave an informal sketch of the semantics behind this syntactic translation. In Section 5 we will give a formal treatment of the related semantical correspondence between Reiter extensions and minimal temporal models, induced by the interpretation mapping τ.

The correspondence we are aiming at will be such that the epistemic state $\mathbf{M_t}$ of a temporal epistemic model $\mathbf{M}$ will be exactly the set of models of $\mathbf{E_t}$ and the limit model will be the set of models of $\mathbf{E}$:

$$\mathbf{M_t} = \mathbf{Mod(E_t)} \qquad \mathbf{E_t} = \mathbf{Th(M_t)}$$

$$\mathbf{lim\ M} = \mathbf{Mod(E)} \qquad \mathbf{E} = \mathbf{Th(lim\ M)}$$

To get the idea we start with an example.

Example 5.1

Let $\Delta = <\mathbf{W}, \mathbf{D}>$ be a default theory for signature $\Sigma = <\mathbf{a, b, c, d, e}>$ defined by

$$\mathbf{W} = \{\ \mathbf{a},\ \mathbf{d},\ \mathbf{b} \rightarrow \neg\, \mathbf{c}\ \}$$

$$\mathbf{D} = \{\ (\mathbf{a} : \mathbf{b}) / \mathbf{b},\ (\mathbf{d} : \mathbf{c}) / \mathbf{c},\ (\mathbf{b} : \neg\, \mathbf{c}) / \mathbf{e}\ \}$$

This default theory has two Reiter extensions:

Firstly, $\mathbf{E} = \mathbf{Cn(\{\ a,\ d,\ b,\ \neg\, c,\ e,\ b \rightarrow \neg\, c\ \})}$ is a Reiter extension:

$$\mathbf{E_0} = \mathbf{Cn(W)}$$

$$\mathbf{E_1} = \mathbf{Cn(E_0 \cup \{b\})} = \mathbf{Cn(\{a,\ b,\ \neg\, c,\ d,\ b \rightarrow \neg\, c\})}$$

$$\mathbf{E_2} = \mathbf{Cn(E_1 \cup \{e\})} = \mathbf{Cn(\{a,\ b,\ \neg\, c,\ d,\ e,\ b \rightarrow \neg\, c\})}$$

$$\mathbf{E_3} = \mathbf{Cn(E_2 \cup \varnothing)} = \mathbf{E_2}$$

$$\mathbf{E_i} = \mathbf{E_2} \quad \text{for all} \quad \mathbf{i > 3}$$

and

$$\mathbf{E} = \bigcup_{\mathbf{i=0}}^{\infty} \mathbf{E_i} = \mathbf{E_2}$$

A second Reiter extension is $\mathbf{F} = \mathbf{Cn\{a,\ d,\ c,\ \neg\, b,\ b \rightarrow \neg\, c\ \}}$:

$$\mathbf{F_0} = \mathbf{Cn(W)}$$

$$\mathbf{F_1} = \mathbf{Cn(F_0 \cup \{c\})} = \mathbf{Cn(\{a,\ \neg\, b,\ c,\ d,\ b \rightarrow \neg\, c\ \})}$$

$$F_2 = Cn(F_1 \cup \varnothing) = F_1$$

$$F_i = F_1 \text{ for all } i > 2$$

and

$$F = \bigcup_{i=0}^{\infty} F_i = F_1$$

The temporal epistemic model **M** which corresponds with **E**, and the model **N** corresponding with **F** are shown in figure 1:

M

a	1 1 1 1 ...
b	u 1 1 1 ...
c	u 0 0 0 ...
d	1 1 1 1 ...
e	u u 1 1 ...

N

a	1 1 1 ...
b	u 0 0 ...
c	u 1 1 ...
d	1 1 1 ...
e	u u u ...

Figure 1

In this picture, time runs from left to right. Only the atoms are shown, where a **1** means the atom is known in M_t, a **0** means that the negation of the atom is known, and a **u** means that neither the atom nor its negation is known (which means that M_t contains a valuation in which the atom is true and one in which it is false). Thus, M_0 contains all valuations **m** for which $m(a) = 1$ and $m(d) = 1$.

It is easy to verify that both **M** and **N** are conservative, and therefore satisfy all rules of the form $C(\alpha) \rightarrow G(\alpha)$ (with α a propositional formula) and that $\tau(W)$ is true at all points. Furthermore, the temporal rules translating the default rules are:

$$Ca \wedge \neg F(\neg b) \rightarrow Gb$$

$$Cd \wedge \neg F(\neg c) \rightarrow Gc$$

$$Cb \wedge \neg Fc \rightarrow Ge$$

Both models satisfy these rules. Moreover, both models minimally satisfy the requirements (with respect to the refinement relation $\leq$ between temporal epistemic models). The correspondence between the Reiter extensions and the epistemic states can be described by

$$\mathbf{E_t = Th(M_t)} \qquad \mathbf{F_t = Th(N_t)}$$

$$\mathbf{E = Th(lim\ M)} \qquad \mathbf{F = Th(lim\ N)}$$

In the following two Propositions 5.2 and 5.3 we will treat the two directions of the correspondence between Reiter extensions of a default theory and minimal temporal models of its temporal interpretation. Of course we can never hope to find a model of an inconsistent extension. Therefore we will assume that the set of axioms of a default theory is consistent, as this ensures that the extensions, if any exists, are consistent.

Proposition 5.2

Let $\Delta = \langle \mathbf{W}, \mathbf{D} \rangle$ be a default theory and $\mathbf{M}$ a minimal temporal model of $\tau(\Delta)$. Then the set $\mathbf{E}$ defined by $\mathbf{E} = \mathbf{Th(lim\ M)}$ is a Reiter extension of Δ. Moreover, $\mathbf{E_t} = \mathbf{Th(M_t)}$ for all $t \in \mathbb{N}$.

Proposition 5.3

Let $\Delta = \langle \mathbf{W}, \mathbf{D} \rangle$ be a default theory with $\mathbf{W}$ consistent and $\mathbf{E}$ a Reiter extension of Δ. Then the temporal epistemic model $\mathbf{M}$ defined by $\mathbf{M} = (\mathbf{Mod(E_t)})_{t \in \mathbb{N}}$ is a minimal temporal model of $\tau(\Delta)$ with $\mathbf{lim\ M} = \mathbf{Mod(E)}$.

We can use the consequence relation $\models_{min}$ to describe sceptical entailment of a default theory:

Definition 5.4 (sceptical consequence)

Given a default theory Δ, by $\mathbb{E}(\Delta)$ we denote the set of its Reiter extensions. A propositional formula φ is called a *sceptical consequence of* Δ, denoted by $\Delta \mid\!\sim_{sc} \varphi$, if $\bigcap \mathbb{E}(\Delta) \models \varphi$ (or, if it is an element of all extensions of Δ).

We are now ready to state our main correspondence result between Reiter extensions and minimal temporal models (relying of course on Propositions 5.2 and 5.3), and the relation between their respective sceptical entailment relations.

Theorem 5.5 (semantic correspondence)

Let $\Delta = < W, D >$ be a default theory of signature Σ with W consistent and let

$$\mathbb{E}(\Delta) = \{ E \mid E \text{ is a Reiter extension of } \Delta \}$$

$$\mathbb{M}(\tau(\Delta)) = \{ M \mid M \text{ is a minimal temporal model of } \tau(\Delta) \}$$

a) By

$$\Phi(E) = (Mod(E_t))_{t \in \mathbb{N}}$$

$$\Psi(M) = Th(\lim M)$$

two bijective mappings

$$\Phi : \mathbb{E}(\Delta) \rightarrow \mathbb{M}(\tau(\Delta))$$

$$\Psi : \mathbb{M}(\tau(\Delta)) \rightarrow \mathbb{E}(\Delta)$$

are defined that are each others inverse.

In other words, the equations

$$M = (Mod(E_t))_{t \in \mathbb{N}}$$

$$E = Th(\lim M)$$

define a one-to-one correspondence between $E \in \mathbb{E}(\Delta)$ and $M \in \mathbb{M}(\tau(\Delta))$.

b) For any propositional formula φ:

$$\Delta \mathrel{\boxed{\,\mid\!\sim\,}}_{sc} \varphi \quad \Leftrightarrow \quad \tau(\Delta) \vDash_{min} F(\varphi)$$

This interpretation yields temporal semantics to default logic: given a default theory Δ, its semantics is given by $\mathbb{M}(\tau(\Delta))$. Note that no minimal models of $\tau(\Delta)$ exist if W is inconsistent (the default theory is classically inconsistent), or if Δ has no extensions (the default theory is nonmonotonically inconsistent). This is similar to other semantics for default logic.

In the literature there exist a number of approaches to giving semantics to default logic. In the next section we will compare them with our temporal approach.

6 Other approaches to semantics for default logic

In this section we will compare our approach with other approaches to semantics for default logic, as known from literature: (Gabbay 1982), (Etherington 1987), (Besnard and Schaub 1994), (Schaub 1991), (Marek and Truszczynski 1989), (Marek and Truszczynski 1992), (Marek, Schwarz and Truszczynski 1993), (Schwarz and Truszczynski 1994), (Schwarz 1995).

Comparison with Gabbay's intuitionistic approach

In (Gabbay 1982) an approach to nonmonotonic logic is described where intuitionistic logic is used as a basis. The semantics are described by Kripke models (in the form of temporal frames) where the accessibility relation is a pre-ordering on the worlds according to time points that describe the stages in the reasoning (a well-known approach to the semantics of intuitionistic logic; see (Kripke 1965)). This idea of using temporal frames to represent the flow of time of the reasoning process itself is in common with our approach. However, there are differences as well. As Gabbay's approach does not use epistemic states, one always has to commit to justifications: it is not possible to express that the truth value of a justification β should be left open in the future of the reasoning process. In our approach there is a choice: either one can choose to commit to justifications or not. The second case is described in the current paper, while a slight modification of the temporal translation of the default rules will enable our approach to commit to justifications. The way this can be worked out is beyond the scope of this paper. A second difference with Gabbay's approach is that we do not give temporal interpretations to classical connectives such as negation and implication, whereas the intuitionistic approach does: e.g., $\neg\alpha$ is true at a time point if and only if for all future time points α is false. A third difference is that in our case there is a time difference (in principle of one step) between the conclusion γ of a default rule $(\alpha : \beta) / \gamma$ and its condition α. In Gabbay's approach both α and γ refer to the same time point, while only β is interpreted in a temporal manner. We interpret both β and γ

in a temporal manner. This essentially means that in Gabbay's approach default reasoning steps are not counted by the time measure as used. This difference has rather far-reaching implications for the models. In Gabbay's case the conclusions of the reasoning process are meant as those statements that are true at all time points of the intended model, whereas in our case they are the statements that are ("become") known to be true at some time point of the model, i.e., that are known to be true in the limit model. Gabbay's logic does not yield semantics for Reiter's default logic. In (Lukaszewicz 1990), pp. 149-154, a critical analysis is given of Gabbay's approach.

Comparison with Etherington's semantics

In (Etherington 1987) it is argued that a semantics of default logic in terms of typical semantic structures as known is not possible, because the outcome of a default reasoning process essentially depends on *the way knowledge is extended* (see p. 497), and this requires knowledge that is not inherent in typical semantical structures. Precisely this view was our motivation to model the traces of the reasoning process explicitly in our semantics. Etherington's semantics has some similarities and some differences with our approach. The main similarity is that our minimal temporal models correspond to (maximal) chains in the sense of Etherington's preference ordering. A maximal element with respect to Etherington's ordering corresponds to our notion of limit model. Actually we define the (temporal) ordering relation between states in the reasoning process in a logical manner by temporal axioms (the temporal translations of the default rules), whereas Etherington gives a more ad hoc definition of his preference relation. Our notion of minimality with respect to the usual refinement relation corresponds to what in Etherington's case is also hidden in the definition of the preference relation, namely that nothing else can happen than what is based on (generated by) the given defaults (a kind of groundedness-condition).

Comparison with Besnard and Schaub

The approach of Besnard and Schaub (Besnard and Schaub 1994) is similar to an earlier approach to semantics for Reiter's default logic by Schaub (Schaub 1991). Instead of pairs of classes of interpretations, one for the formulas in an extension, one for the justifications, Besnard and Schaub use classes of Kripke models with one *actual world*, where the formulas of an extension have to be true in the actual worlds, and the worlds reachable from the actual world are used for the justifications. Also an ordering $<_{D'}$ is defined on classes of Kripke models, which depends on the defaults in the default theory. Although both in their approach and ours, Kripke models are used, the way in which they are used is quite different, not to mention the fact that we are using epistemic states, and Besnard and Schaub are using two-valued models for the extensions. As in Etherington's approach, maximal chains in their ordering correspond to our minimal temporal models, and Besnard and Schaub also give a more ad hoc definition of their precedence relation. As we want our models to reflect the reasoning path which leads to an extension, it was natural to use a linear time model, and as at any point in time, not all facts will be known when reasoning, the use of epistemic states seems justified. Both their approach and ours use minimisation of models with respect to a preference relation. Their ordering $<_{D'}$ depends on the default theory, whereas our ordering $\leq$ is structurally defined, independent of the defaults.

Comparison with Marek, Schwarz and Truszczynski

There is a long tradition of research into modal nonmonotonic logics starting with (McDermott and Doyle 1980). With every modal logic of knowledge (of belief) one can associate a nonmonotonic logic based on it. Given a theory I in the modal language, an expansion T is a theory in this language satisfying a certain fixpoint definition. These expansions play a role similar to the role extensions play in default logic. For a number of modal logics (most notably a logic called S4F), it is possible to translate a default theory Δ into a theory I_{Δ} such that expansions of I_{Δ} correspond to extensions of Δ. The translation of a default rule is quite similar to our definition: a rule $(\alpha : \beta) / \gamma$ is translated into the modal formula $L\alpha \wedge LM\beta \rightarrow \gamma$, where $M = \neg L \neg$ by definition (see for instance (Marek, Schwarz and Truszczynski 1993)). This rule is to be read as: if you know α and you know β is possible, then γ is true. The essential difference with our approach is that this modal rule can be seen as a static (closure) condition on the beliefs of the agent. Any set of beliefs that can be regarded as the set of beliefs of a rational introspective agent (that is, it must be an expansion), must be closed under the default rules. Our translation emphasises the dynamic (behavioural) aspect. A minimal temporal epistemic model does not describe a belief set of an agent, but a reasoning trace of a rational introspective agent. The limit of such a model corresponds to a (final) belief set. Thus, our logic does not fall into the general framework of Marek, Truszczynski and Schwarz. Our underlying (monotonic) logic, temporal epistemic logic, is essentially just standard S5, with a straightforward temporalization over the natural numbers. So we use simpler techniques than the approach of (Marek, Schwarz and Truszczynski 1993) which is based on S4F (or a modal logic in between the logics N and S4F. In (Amati, Aiello, Gabbay and Pirri 1996) it is shown that, with a slight adaptation of the fixpoint equation and the translation, logics between KD4 and KD4Z are also suited).

Comparison with Amati, Aiello and Pirri

The idea of Marek, Schwarz and Truszczynski is taken even further in (Amati, Aiello and Pirri 1996), where it is shown that extensions correspond to certain theorems in the modal logic KD4Z. The fixpoint for extensions is expressible in the language. Thus, the fixpoint is not a construction on top of the logic, but extensions correspond to fixpoints expressed in the language, provable in KD4Z. Again, however, this is a static description of the set of beliefs of an agent. Our perspective is different: we want to make the construction an explicit temporal process as performed by the agent.

In (Lin and Shoham 1992) a bimodal logic of knowledge and assumptions is described. This logic has two modal operators, $\mathbf{K}$ for knowledge and $\mathbf{A}$ for assumptions. A preference relation on models for this logic favors models with less knowledge and the same assumptions. Preferred models are models which are minimal in this ordering, with the extra condition that the knowledge must be equal to the assumptions. Via a translation of default rules which maps a rule $(\alpha : \beta) / \gamma$ into the formula $\mathbf{K}\alpha \wedge \neg \mathbf{A} \neg \beta \rightarrow \mathbf{K}\gamma$ they get a representation result similar to ours (preferred models of the translation correspond to extensions). In fact, for a restricted version of their language (expressing some sort of rules) they have a characterization result of minimal models analogous to the characterization of extensions of Lemma 3.1. Roughly, it states that the knowledge of a minimal model is equal to the union of an increasing sequence of sets of formulae, where a set in the sequence is the result of applying some rules to the previous set. In this fashion, a minimal model in their sense is very similar to a minimal temporal model in our sense. The condition $\neg \mathbf{A} \neg \beta$ in the translation of a default rule is fulfilled in a preferred model just in case $\neg \beta$ is not in the knowledge of the preferred model (since in a preferred model, assumptions coincide with knowledge). The knowledge of such a preferred model corresponds to the knowledge of the limit of a minimal model in our approach, and $\neg \beta$ is not in the limit just in case $\neg \mathbf{F} \neg \beta$ is true in the temporal model. The difference between our approach and theirs is again in the perspective: their perspective is static (describing a final set of beliefs), with a characterization result that implicitly gives a dynamic description. In our approach, the dynamic perspective is the most important, and the static notion of limit is derived from the dynamic notions.

7 Conclusions and further work

In this paper we have given a temporal interpretation to the notion of a justification in a default rule. This led us to an interpretation mapping of default theories into temporal theories, and to a one-to-one correspondence between the Reiter extensions of such a default theory and the minimal temporal epistemic models of its temporal interpretation. This work enables one to use concepts from temporal logic to study default reasoning. Of course such a translation does not automatically imply that the problems of default logic will be solved at once. The temporal epistemic logic we designed has its own complexity. Nevertheless, both this temporal epistemic logic, and its connection to default logic seem worth to be investigated further.

The interpretation and correspondence yield a temporal semantics for default logic. Although there are other approaches in the literature for giving semantics to default logic, we feel that making the temporal aspect explicit in a formalism where the dynamics of the reasoning process (choosing the default assumptions) have an impact on the final outcome, gives a clear and intuitively appealing meaning to default logic.

In (Engelfriet and Treur 1996) we have shown that instead of linear time models, we can also use branching time models for describing default logic. For a large class of default theories it is possible to find a "largest" branching time model which contains all the extensions as branches. In such a model the points in time where a branching takes place are exactly the points where a conflicting choice between some default assumptions has to be made.

In (Hoek, Meyer, Treur 1995) Temporalised Epistemic Default Logic (TEDL) is introduced. A similarity with the approach introduced in the current paper is that a dynamic perspective on default reasoning is used. However, there are two differences. The first difference is that TEDL formalises a default logic quite different from Reiter's Default Logic: in TEDL the justifications refer only to the current state of knowledge; knowledge that is acquired at later points in time is not taken into account. The notion of extension, or final conclusion set, is defined in a constructive manner; no fix point definition is used. This implies that the conclusion sets are different from Reiter extensions: a TEDL-conclusion set E may be based on (generated by) consequents of default rules for which the prerequisite is included in E, but the negation of the justification is also in E. A second difference is that the semantics of TEDL is defined by labelled branching time temporal models.

Appendix

In this Appendix we will give proofs of the main propositions.

Proposition 5.2

Let $\Delta = <W, D>$ be a default theory and M a minimal temporal model of $\tau(\Delta)$. Then the set E defined by $E = Th(\lim M)$ is a Reiter extension of Δ. Moreover, $E_t = Th(M_t)$ for all $t \in \mathbb{N}$.

Proof

Let M be a minimal temporal model of $\tau(\Delta)$ and $E = Th(\lim M)$. Let the sets E_t be defined as in Lemma 3.1, (i).

a) We will first show that $E_n = Th(M_n)$ for all $n \in \mathbb{N}$, by induction on n.

\- *Initial step* $n = 0$:

Since $(M, 0) \models \tau(W)$ we have $Th(M_0) \supseteq Cn(W) = E_0$. To prove equality, construct a temporal epistemic model M' with $M'_0 = Mod(W)$ and $M'_t = M_t$ for all $t > 0$. It is easy to see that $M' \models \tau(\Delta)$ and $M' \leq M$ and as M was a minimal model of T_Δ we have $M' = M$ so $Th(M_0) = Th(M'_0) = Cn(W) = E_0$.

\- *Induction step*:

We have to prove that $E_{n+1} = Th(M_{n+1})$.

Choose a $\varphi \in E_{n+1} = Cn(E_n \cup \{ \omega \mid (\alpha : \beta) / \omega \in D, \alpha \in E_n, \neg\beta \notin E\})$, that is

$$E_n \cup \{ \omega \mid (\alpha : \beta) / \omega \in D, \alpha \in E_n, \neg\beta \notin E\} \models \varphi.$$

Take a valuation $m \in M_{n+1}$. Then since $M_{n+1} \subseteq M_n$ (conservativity because $M \models C'$) we have $m \in M_n$ and as (induction hypothesis) $E_n = Th(M_n)$ we have $m \models E_n$. Now consider a formula ω with $(\alpha : \beta) / \omega \in D, \alpha \in E_n$ and $\neg\beta \notin E$. As $\alpha \in E_n$ and $E_n = Th(M_n)$ we have $(M, n) \models C\alpha$. Furthermore, $\neg\beta \notin E = Th(\lim M) = \bigcup_{t=0}^{\infty} Th(M_t)$ (see Lemma 2.7), so $M_i \not\models \neg\beta$ for all i in $\mathbb{N}$, so

$(M, n) \models \neg F(\neg \beta)$. Since $M \models \tau(\Delta)$ we have $(M, n) \models C\alpha \wedge \neg F(\neg \beta) \rightarrow G\gamma$ so $M_{n+1} \models \omega$ so $m \models \omega$. As $m \models E_n \cup \{ \omega \mid (\alpha : \beta) / \omega \in D, \alpha \in E_n, \neg \beta \notin E\}$ we have $m \models \varphi$. We have proved $\varphi \in Th(M_{n+1})$ so $E_{n+1} \subseteq Th(M_{n+1})$.

To prove that they are equal, define the temporal epistemic model M', with $M'_i = M_i$ for $i \neq n+1$ and $M'_{n+1} = Mod(E_{n+1})$. Since $E_{n+1} \subseteq Th(M_{n+1})$ it is easy to see that $M' \models C' \cup \tau(W)$ and $M' \leq M$.

Let $t \in \mathbb{N}$, and $C\alpha \wedge \neg F(\neg \beta) \rightarrow G\gamma \in \tau(D)$ be given, and suppose $(M', t) \models C\alpha \wedge \neg F(\neg \beta)$, so $M'_t \models \alpha$ and $M'_i \not\models \neg \beta$ for all $i > t$. If $t > n$ then it is easy to see that $(M, t) \models C\alpha \wedge \neg F(\neg \beta)$ so $(M, t) \models G\gamma$ and as $M'_i = M_i$ for $i > n + 1$ we have $(M', t) \models G\gamma$, so $(M', t) \models C\alpha \wedge \neg F(\neg \beta) \rightarrow G\gamma$. So suppose $t \leq n$.

Since $M'_i = M_i$ for $i \neq n+1$ we have $(M, t) \models C\alpha$, and $M_i \not\models \neg \beta$ for all $i \neq n+1$, but then also $M_{n+1} \not\models \neg \beta$ because of conservativity of M (since $M \models C'$). So $(M, t) \models C\alpha \wedge \neg F(\neg \beta)$ and thus $(M, t) \models G\gamma$ so $M'_i \models \gamma$ for $i > t$ and $i \neq n + 1$. Since $M_t \models \alpha$ and by induction $E_t = Th(M_t)$ we have $\alpha \in E_t$. Furthermore, $M_i \not\models \neg \beta$ for all i, so $\lim M \not\models \neg \beta$ so $\neg \beta \notin E$. Since $(\alpha : \beta) / \gamma \in D$ we have $\gamma \in E_i$ for $i > t$ so in particular $\gamma \in E_{n+1} = Th(M'_{n+1})$. It follows that $M'_i \models \gamma$ for $i > t$ so $(M', t) \models G\gamma$ and $(M', t) \models C\alpha \wedge \neg F(\neg \beta) \rightarrow G\gamma$.

In both cases we have $M' \models \tau(D)$, so $M' \models \tau(\Delta)$. As $M' \leq M$ and M is a minimal model of T_Δ we have $M = M'$ so $E_{n+1} = Th(M'_{n+1}) = Th(M_{n+1})$, which was to be proven.

As $E_t = Th(M_t)$ for all $t \in \mathbb{N}$, we have:

$$E = Th(\lim M) = \bigcup_{t=0}^{\infty} Th(M_t) = \bigcup_{t=0}^{\infty} E_t$$

and therefore E is an extension in Reiter's sense.

∎

Proposition 5.3

Let $\Delta = \langle W, D \rangle$ be a default theory with W consistent and E a Reiter extension of Δ. Then the temporal epistemic model $\mathbf{M}$ defined by $\mathbf{M} = (\mathbf{Mod}(\mathbf{E_t}))_{t \in \mathbb{N}}$ is a minimal temporal model of $\tau(\Delta)$ with $\lim \mathbf{M} = \mathbf{Mod}(\mathbf{E})$.

Proof

Let Δ, $\mathbf{E}$ and $\mathbf{M}$ be given as above. It is easy to see that $\mathbf{M}$ is a temporal epistemic model. First we shall prove that $\mathbf{M}$ is a model of $\tau(\Delta)$, after which the minimality of $\mathbf{M}$ will be shown.

Since $\mathbf{E_t} \subseteq \mathbf{E_{t+1}}$ and $\mathbf{W} \subseteq \mathbf{E_t}$ for all $\mathbf{t}$, it is easy to see that $\mathbf{M} \models \mathbf{C'} \cup \tau(\mathbf{W})$.

Now take an arbitrary rule $\mathbf{C}\alpha \wedge \neg \mathbf{F}(\neg \beta) \rightarrow \mathbf{G}\gamma \in \tau(\mathbf{D})$, any $\mathbf{t} \in \mathbb{N}$ and assume that

$$(\mathbf{M}, \mathbf{t}) \models \mathbf{C}\alpha \wedge \neg \mathbf{F}(\neg \beta).$$

Then $\mathbf{M_t} \models \alpha$ and $\forall \mathbf{s} > \mathbf{t}$: $\mathbf{M_s} \not\models \neg \beta$, so we have a default rule $(\alpha : \beta) / \gamma \in \mathbf{D}$, with $\alpha \in \mathbf{E_t}$ and there is no $\mathbf{s} > \mathbf{t}$ with $\neg\beta \in \mathbf{E_s}$, and therefore there is no $\mathbf{s} \in \mathbb{N}$ with $\neg\beta \in \mathbf{E_s}$ and consequently $\neg\beta \notin \mathbf{E}$. It follows that $\gamma \in \mathbf{E_{t+1}}$ and thus that $\forall \mathbf{s} > \mathbf{t}$: $\gamma \in \mathbf{E_s}$, and therefore that $\forall \mathbf{s} > \mathbf{t}$: $\mathbf{M_s} \models \gamma$, i.e., $(\mathbf{M}, \mathbf{t}) \models \mathbf{G}\gamma$. Therefore $(\mathbf{M}, \mathbf{t}) \models \mathbf{C}\alpha \wedge \neg \mathbf{F}(\neg \beta) \rightarrow \mathbf{G}\gamma$. As the rule and the time point were chosen arbitrarily, $\mathbf{M} \models \tau(\mathbf{D})$, hence $\mathbf{M} \models \tau(\Delta)$.

Considering the limit case, as $\mathbf{M}$ is conservative, $\lim \mathbf{M}$ exists and

$$\lim \mathbf{M} = \bigcap_{t=0}^{\infty} \mathbf{M_t}$$

$$= \bigcap_{t=0}^{\infty} \mathbf{Mod}(\mathbf{E_t})$$

$$= \mathbf{Mod}(\bigcup_{t=0}^{\infty} \mathbf{E_t})$$

$$= \mathbf{Mod}(\mathbf{E})$$

Minimality

We now turn to the minimality of $\mathbf{M}$. Suppose $\mathbf{M}$ is not minimal, then there exists a model $\mathbf{M'}$ with $\mathbf{M'} \models \tau(\Delta)$, $\mathbf{M'} \leq \mathbf{M}$ and $\mathbf{M'} \neq \mathbf{M}$. There must be a smallest time point $\mathbf{n} \in \mathbb{N}$ for which $\mathbf{M'_n} \neq \mathbf{M_n}$. We shall distinguish between two cases:

(i) $n = 0$: Then $Cn(W) = E_0 = Th(M_0)$ which is a proper superset of $Th(M'_0)$, so there exists a formula $\alpha \in W$ such that $M'_0 \not\models \alpha$ so $(M', 0) \not\models C\alpha$ so $M' \not\models \tau(W)$, in contradiction with $M' \models \tau(\Delta)$.

(ii) $n > 0$: Then $M'_i = M_i = Mod(E_i)$ for all $i < n$. Take a formula $\varphi \in Th(M_n) \setminus Th(M'_n)$. Then

$$E_{n-1} \cup \{ \omega \mid (\alpha : \beta) / \omega \in D, \alpha \in E_{n-1}, \neg \beta \notin E\} \models \varphi.$$

Now take a valuation $m \in M'_n$, then $m \in M'_{n-1}$ and since $M'_{n-1} = M_{n-1}$ and $M_{n-1} \models E_{n-1}$ we have $m \models E_{n-1}$. Now take an ω with a default rule $(\alpha : \beta) / \omega \in D$ such that $\alpha \in E_{n-1}$ and $\neg \beta \notin E$. Then, since $M'_{n-1} = M_{n-1}$ we have $M'_{n-1} \models \alpha$ so $(M', n-1) \models C\alpha$. Furthermore, $\neg \beta \notin E$ so $\lim M \not\models \neg \beta$ so $M_t \not\models \neg \beta$ for all t and since $M' \leq M$ we have $M'_t \not\models \neg \beta$ for all t, so $(M', n-1) \models \neg F(\neg \beta)$. As $C\alpha \wedge \neg F(\neg \beta) \rightarrow G\omega \in \tau(D)$ and $M' \models \tau(D)$, by $(M', n-1) \models C\alpha \wedge \neg F(\neg \beta)$ we have $(M', n-1) \models G\omega$ so $M'_n \models \omega$. As $m \in M'_n$ we get $m \models \omega$. So $m \models E_{n-1} \cup \{ \omega \mid (\alpha : \beta) / \omega \in D, \alpha \in E_{n-1}, \neg \beta \notin E\}$ and then $m \models \varphi$. We have proven that $\varphi \in Th(M'_n)$, in contradiction with the assumption.

In both cases we get a contradiction, so we have to conclude that such a model M' cannot exist, and therefore that M must be minimal.

∎

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List of contents

List of figures