Towards Performance Evaluation of Mobile Ad Hoc Network Protocols

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***Abstract*—We present a formal framework to evaluate stochastic properties of MANET protocols. It captures the interplay between stochastic behavior of protocols deployed at different network layers, and the underlying dynamic topology. The link connectivity model, which implicitly models node mobility, specifies link up and down lifetimes. We use so-called *constrained labeled multi-transition systems* (CLMSs) to specify MANETs; transitions are annotated by *network restrictions*, capturing the topologies in which a transition is possible. A continuous Markov chain can be generated from a CLMS, to evaluate the performance of the corresponding MANET.**

I. Introduction

In mobile ad hoc networks (MANETs), nodes communicate with each other using wireless transceivers (possibly along multihop paths) without the need for a fixed infrastructure. Since wireless communication is unreliable, MANET protocols should be able to tolerate faults that may arise due to unreliable communication.

Performance evaluation of MANET protocols depends on physical characteristic of nodes, the underlying topology and its dynamism, and the protocol behavior itself and its collaboration with other protocols at different network layers. The physical characteristics of nodes and their underlying topology define whether two nodes can communicate effectively. For MANET protocols above the data-link layer, one of the most important factors that affect their performance are data-link layer protocols.

MANET protocols are usually evaluated by means of simulation: a network of nodes is modeled and then run for a set of scenarios in a specific simulation environment. In each scenario, the set of events generated by the nodes are specified. The simulation environment may take into account the physical area in which nodes are located, the time duration of simulation, the physical characteristics of nodes, and a node mobility model, which defines the speed and direction of a node's movement over time. Formal methods offer an interesting alternative to model such networks, and evaluate them using (semi-)automated tools.

In this paper we aim at providing a framework to evaluate the performance of MANET protocols above the data-link layer regarding physical characteristic of nodes, the underlying topology, node mobility, and data-link protocols. Key parameters are the data-link layer response time, its message queue capacity, the probability that a node receives a message (P_{rcv}), and a mobility model. A mobility model is captured by the link connectivity model which is characterized by means of link up or down lifetimes. By this framework we can evaluate responsiveness of a MANET protocol deployed on a specific data-link layer in a dynamic network and measure how quickly a MANET protocol reaches its goals (like information dissemination) and how these parameters effect on its performance. We provide so-called *constrained labeled multi-transition systems* (CLMSs) to specify MANETs; transitions are annotated by *network restrictions*, capturing the topologies in which a transition is possible. The rates of transitions are adjusted regarding physical layer issues, data-link layer and mobility of nodes. A continuous Markov chain can be generated from a CLMS, to evaluate the performance of the corresponding MANET using e.g. a probabilistic model checker. We apply this approach to a protocol that uses flooding to determine the maximum identifier in a MANET.

II. Concepts for Modeling MANETs

The main modeling concepts of MANETs are wireless communication and the dynamism of the underlying topology. Nodes are equipped with wireless transceivers, by which they send and receive information. For each (sender) node, a transmission range is considered, which is an area in which the strength of emitted signals (of data) from the node is strong enough to be sensed by other nodes. The transmission range is not the same for different nodes, and depends on the power used to emit the signal.

A. Wireless Communication

MANETs have a modular (or layered) design. A layer is a collection of conceptually similar functions that provides services to the layer above it and receives services from the layer below it. The *Open System Interconnection (OSI) Reference Model* is an abstract description for layered communication and computer network protocols. In its most basic form, it divides network protocols into seven layers which, from top to bottom, are the application, presentation, session, transport, network, data-link, and physical layers. Protocols at the physical layer primarily concern the interaction of a single device with a medium, while protocols at the data-link layer concern interactions of multiple devices with a shared medium. The *Media Access Control* sub-layer (MAC) manages the access of several network nodes to a shared transmission medium and resolves their contention. In this paper we focus on performance evaluation of protocols above the data-link layer. Thus our formal framework should embed the services of lower layers (data-link and physical layers) in its syntax and semantics, as well as wireless communication provided by the data-link layer.

In a MANET, when a node broadcasts, all nodes located in its transmission range are potential receivers. This *local* broadcast is non-blocking and lossy, which means the sender delivers its message to its data-link layer irrespective of who is going to receive, and a receiver may lose the message due to signal interference. If two nodes with a common node in their transmission range broadcast simultaneously, the emitted signals may interfere at the receiver. MAC sub-layer protocols, at each node, are responsible to prevent such interferences in MANETs. However, interferences can not be avoided completely; for instance, IEEE 802.11 exploits a schema to reduce interferences, but does not offer any recovery on broadcast frames. Generally there are two types of local broadcast implemented in the data-link layer, namely *unreliable* and *reliable* (including unicast communication). In unreliable local broadcast, each message broadcasts once, while in reliable local broadcast, each message is transmitted several times. The duration of an unreliable broadcast is equal to the duration a node needs to wait to obtain the medium, and this duration (called average service time T_s) can be abstracted by the average response time T_{Mac}. The average response time is the average time a message spends in a data-link layer queue, either waiting (T_d) to be transmitted or being transmitted (T_s). For instance, the average response and service time for unreliable broadcast in the IEEE 802.11 MAC protocol is modeled in [1].

Let Loc denote a finite set of addresses, ranged over by ℓ, which represent hardware addresses of nodes. Since transmission is lossy, a node ℓ will receive the data with some probability P_{rcv_ℓ}. In [2], this probability is computed by taking the distance, signal strength and interference of other nodes into account. Thus this parameter provides an abstraction from the effects of physical characteristics of nodes and their interferences.

To provide a suitable modeling framework for the evaluation of protocols above the data-link layer while the functionalities of the MAC sub-layer are embedded, we consider unreliable local broadcast in our framework, and we abstract away from collisions and only consider successful receive actions. Therefore we can interleave concurrent local broadcasts regarding successful receives. We model the data-link layer as a queue with a bounded capacity, and an average response time T_{Mac}. Hence when a node broadcasts, if the data-link layer queue is not full, it gets the message and on average transmits it after T_{Mac} time units. If a node is located in the communication range of the sender, it will receive the data with probability P_{rcv}.

A, B, C denote concrete addresses. We say that a node B is *connected to* a node A, if A is located within the transmission range of B. It is said that A is in the vicinity of B. For instance, in Fig. 1, nodes A and C are located in the transmission range of B. Hence they can receive data from B. This connectivity relation between nodes, which is not necessarily symmetric, introduces a topology concept.

A topology is a function $\gamma : Loc \rightarrow I\!\!P Loc$, where $\gamma(\ell)$ denotes the set of nodes that ℓ is connected to. This function models unidirectional connectivity between nodes. For instance in Fig. 1, the underlying topology is $\gamma(A) = \emptyset$, $\gamma(B) = \{A, C\}$, $\gamma(C) = \{B\}$.

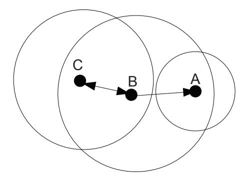

Figure 1. The transmission range of each node is indicated by a circle: nodes B and C are connected to each other, while B is connected to A.

B. Mobility

Since nodes in MANETs are mobile, the underlying topology changes. There are two approaches to modeling topology changes. One models the underlying topology explicitly as a part of the semantics; then mobility is modeled by internal transitions to states with a different network topology. In the other approach, mobility is modeled implicity; each state is representative of all possible topologies (under a mobility model) a network can meet, and a network can be at any of these topologies.

We model mobility implicitly in the semantics following the approach given in [3], [4]. In this approach each state is representative of all possible topologies (under a mobility model) and each transition is subscripted by a set of topologies for which such behavior can be observed. This approach results in a compact labeled transition system and releases us from encoding the underlying topology as part of the state. We introduced network restrictions in [4] to formally specify the set of possible topologies after a synchronization. We assume a binary relation $\rightsquigarrow$ on $Loc \times Loc$, which imposes connectivity relations between addresses. A relation $B \rightsquigarrow A$ denotes that a node with address B is connected to a node with address A, meaning that A can receive data from B. A *network restriction* is a set of relations $\ell \rightsquigarrow \ell'$. We write $B \rightsquigarrow A, C$ for $\{B \rightsquigarrow A, B \rightsquigarrow C\}$. The empty network restriction $\{\}$ denotes all possible topologies. Let $\mathcal{C}(\ell)$ denotes the set of nodes that ℓ is connected to. Each network restriction $\mathcal{C}$ represents a set of topologies that satisfy the relations in $\mathcal{C}$, i.e. $\forall \gamma \in \mathcal{C} \cdot \mathcal{C}(\ell) \subseteq \gamma(\ell)$.

A mobility model may restrict the set of topologies a network can meet, and some topologies may occur with higher probability. To evaluate MANET protocols regarding network topology and mobility of nodes, we compute the probability of the (topologies in) $\mathcal{C}$ under a mobility model.

Node mobility follows a mobility behavior/model. There are several mobility models used in the evaluation of MANET protocols [6]. The *random waypoint model* is one the most commonly used mobility models for simulations of MANETs. In this model, each node selects a random destination, uniformly distributed within the two-dimensional space, and then moves toward the destination at a speed that is uniformly distributed between the minimum and maximum speed. When a node reaches its destination, it pauses for a constant pause time. After the pause time expires, a new destination is selected, and the node again moves toward the destination at a random velocity.

Node mobility, coupled with physical layer characteristics, determines the status of link connections and, hence, the dynamic topology. To abstract from node mobility behavior and physical layer issues, following [5], we exploit a link connectivity model which implicitly models mobility behavior. The link connectivity model is a two-state Markov chain (i.e. UP and DOWN states) to model the link between each pair of nodes. The link UP lifetime and link DOWN lifetime are random variables. We denote the mean values of the link UP and DOWN lifetimes between nodes A and B as $T_{AB,UP}$ and $T_{AB,DOWN}$, respectively. In our model, we assume that the link UP and DOWN lifetimes are exponentially distributed random variables. For example, in [5] a network of 10 nodes in a 100×100 square was analyzed using the random waypoint mobility model. The radio range was fixed at 20 units, the maximum node velocity at 20 units per second, and the pause time at 5 sec. The average link UP and DOWN lifetimes were approximated using simulations: 21 and 120 sec. respectively. To achieve the same steady-state average values for the link UP and DOWN lifetimes for the random waypoint model (with the above parameters), we let $T_{BA,UP} = 21$ sec. and $T_{BA,DOWN} = 120$ sec. Hence we can compute $P(BA = UP)$ using the two-state link connectivity model, shown in Fig. 2, by $\frac{21}{21+120} \approx 0.15$. We can model different types of node mobility models considered in the study of MANETs by defining these two parameters for each pair of nodes.

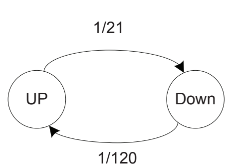

Figure 2. A two-state link connectivity model.

Since nodes are mobile and their mobility is independent of each other, the status of links between each arbitrary pair is independent of others. Consequently we can compute the probability $P(\gamma)$ of each possible topology γ in the steady-state of a mobility model using the two-state Markov chains of each pair of nodes. The probability of γ in Fig. 1 is $P(BA = UP) \times P(BC = UP) \times P(CB = UP) \times P(AB = DOWN) \times P(AC = DOWN) \times P(CA = DOWN)$.

C. Interplay of Network Layers and Mobility

To evaluate MANET protocols, there are many factors to take into consideration: network topology, network traffic, node mobility, and physical layer issues, including the radio frequency channel, terrain, antenna properties, and, perhaps, energy and battery characteristics. Consequently to evaluate a MANET behavior under a network restriction $\mathcal{C}$, we consider the effect of network layers and node mobility: we compute the probability of synchronization induced by $\mathcal{C}$ regarding physical layer issues and interferences of other nodes under a mobility model denoted by $P_r(\mathcal{C})$.

To evaluate a MANET protocol regarding physical layer issues and interferences of other nodes, we compute the probability of a synchronization between a sender and a set of receivers. Let ℓ be the address of a sender and r the set of possible receivers; when a message m is sent, the set of nodes ready to receive such a message are the possible receivers. A possible receiver can successfully receive with probability P_{rcv}, if it is in the range of transmitter. Let $s \subseteq r$ be the set of nodes that receive m (synchronize). Let $P(\ell, s|\gamma, r)$ denote the probability that nodes in s are synchronized to ℓ when the underlying topology is γ and possible receivers are r. We have $P(\ell, s|\gamma, r) = \prod_{\ell' \in Loc} P(\ell'|\ell, s, \gamma, r)$ where:

$$P(\ell'|\ell, s, \gamma, r) = \begin{cases} P_{rcv_{\ell'}} & \ell' \in \gamma(\ell) \cap r \wedge \ell' \in s \\ 1 - P_{rcv_{\ell'}} & \ell' \in \gamma(\ell) \cap r \wedge \ell' \notin s \\ 1 & \ell' \notin \gamma(\ell) \wedge \ell' \notin s \\ 0 & \text{otherwise} \end{cases}$$

When a node ℓ' is in the transmission range of node ℓ such that it is a possible receiver ($\ell' \in \gamma(\ell) \cap r$), then it successfully synchronizes with the sender ($\ell' \in s$) with probability $P_{rcv_{\ell'}}$, as indicated by the first case, or it loses the data ($\ell' \notin s$) with probability $1 - P_{rcv_{\ell'}}$ as indicated by the second case. The third and fourth cases are straightforward.

Suppose the transmission of message m by a node with address ℓ in a MANET is possible for set of topologies represented by $\mathcal{C}$. Each $\mathcal{C}$ induces a synchronization between ℓ and $\mathcal{C}(\ell)$. Thus $P(\ell, \mathcal{C}(\ell)|\gamma, r)$ denotes the probability of a synchronization induced by the network restriction $\mathcal{C}$ regarding physical layer issues and interferences of other nodes, when the underlying topology is γ with possible receivers r. Consequently $\sum_{\gamma \in \mathcal{C}} P(\gamma) \times P(\ell, \mathcal{C}(\ell)|\gamma, r)$ computes the probability of a synchronization induced by the network restriction $\mathcal{C}$ regarding physical layer issues and interferences of other nodes under a mobility model.

Let Γ and $\mathbb{C}$ denote the set of possible topologies and network restrictions over $Loc \rightarrow \mathbb{P}Loc$ and $Loc \times Loc$ respectively. The triple $(\Gamma, \mathbb{C}, P_r)$ constitutes a probability space where $P_r : \mathbb{C} \rightarrow [0, 1]$ assigns $\sum_{\gamma \in \mathcal{C}} P(\gamma) \times P(\ell, \mathcal{C}(\ell)|\gamma, r)$ to each $\mathcal{C}$ regarding the set of possible receivers specified by r. For instance, the probability of $B \rightsquigarrow A$ for possible receivers $s = \{A, C\}$, is computed as follows:

$$P_{\{A,C\}}(B \rightsquigarrow A) =$$
$$P_{rcv_A} \times (1 - P_{rcv_C}) \times P(BA = UP) \times P(BC = UP) +$$
$$P_{rcv_A} \times P(BA = UP) \times P(BC = DOWN)$$

where P_{rcv_A} and P_{rcv_C} result from the data-link abstraction, and $P(BA = UP)$, $P(BC = UP)$ and $P(BC = DOWN)$ result from the mobility model.

III. The Formal Model

Constrained labeled transition systems were introduced in [4] for giving an operational semantics to Restricted Broadcast Process Theory, a process algebra for the specification and verification of MANETs. Here we add concepts from Extended Markovian Process Algebra (EMPA) semantics, a stochastic process algebra [7]. The resulting *constrained labeled multi-transition systems* (CLMSs) can be used to stochastically specify MANETs. The transitions in a CLMS are annotated by network restrictions to indicate the set of possible topologies under which such behavior is possible. Before giving the formal definitions of our framework, we first explain the notations used in our definitions.

To specify data-dependent behavior of MANET protocols, we consider a typed data domain D, with a set of typed operations to specify data terms. D should contain the (distinct) booleans $true$ and $false$. Let V denote a set of (typed) data variables ranged over by x, y. Let u range over close data terms, and w over open data terms. Following the approach of abstract data types [8], we use a set $I\!D$ of equations to represent the semantics for data terms: two data terms u_1 and u_2 are equal if and only if they are equal in the data semantics, denoted by $I\!D \models u_1 = u_2$. An *assignment* is of the form $x := w$, where $x \in V$ and w belong to the same data type. An environment is a well-typed mapping $\theta : X \to D$, where $X \subseteq V$. Θ denotes the set of all environments over V and D. We use $\widehat{w}$ to denote a finite sequence $w_1, \ldots, w_k$, $\widehat{x} := \widehat{w}$ for a sequence of assignments $\{x_1 := w_1, \ldots, x_k := w_k\}$, $w\{\widehat{u}/\widehat{x}\}$ for a sequence of substitutions $\{u_1/x_1, \ldots, u_k/x_k\}$ in w, and $w\{\theta\}$ for $w\{u/x | (x, u) \in \theta\}$, which can be extended to a sequence of data terms by $\widehat{w}\{\theta\}$.

Let M denote a set of message types communicated over a network and ranged over by m; each message type has a set of parameters of type D and is declared by $m(D_1, \ldots, D_n)$. The finite set $dom_m : I\!P(D_1 \times \ldots \times D_n)$ defines the set of possible assignments to the message parameters of m.

Following the approach in [4], we define our framework in two levels: process and network level. At process level, we define the behavior of a process. A MANET consists of a number of nodes that each deploy a process to run. There are two types of actions performed by a process: sending or receiving an action, denoted by $m(\widehat{w})!$ and $m(\widehat{w})?$, respectively. A rate is assigned to each action to indicate the speed at which the action occurs (from the viewpoint of a data-link layer). According to the rates, there are two types of actions: *active* and *passive*:

- *Active* actions define the rate of synchronization, and their rate is a positive real number, interpreted as the parameter of the exponentially distributed random variable specifying the duration of the action.
- *Passive* actions whose rate, denoted by $\perp$, is undefined. The duration of a passive action is fixed only by synchronizing it with an active action.

All send actions are active while receive actions are passive.

Definition 1: A process is specified as an *extended action Markov chain* $\langle S, X, M, \rightarrow, s_0, \zeta_0 \rangle$ over a data domain D:

- S is a set of states.
- $X \subseteq V$ is a set of local variables for the process.
- M is a finite set of message declarations that can be sent or received by the process.
- $\rightarrow$ is a set of transitions. A transition is a tuple $(s, (\alpha, r), s', \langle g, \zeta \rangle)$ where:
  - $s, s' \in S$ are the source and target state.
  - α is a send/receive action, and $r \in I\!R^{\geq 0} \cup \{\perp\}$ its corresponding rate.
  - g, a Boolean data term, is the transition guard.
  - ζ is a set of simultaneous assignments of the form $\widehat{x} := \widehat{w}$, where the $x_i \in X$ are pairwise distinct.
- $s_0 \in S$ is the initial state.
- ζ_0 is the set of initial assignments of local variables to closed data terms, i.e. $x := u$, $\forall x \in X$.

Example. As running example we consider a protocol in which the nodes use a flooding approach to find out the maximum identifier in the network. Hereafter we call this protocol the *Max protocol*. Each node broadcasts its knowledge about id regularly by sending the message $know(id)$ with a rate r. When a node receives a $know(x)$ message from a node in its vicinity, it updates its knowledge if $x > id$ and then continues its broadcast. If $x \leq id$, it broadcasts with a smaller speed (since the nodes in its vicinity know its id with a high probability). Let Nat denote the natural numbers data type. The specification of this protocol for a node with initial identifier n is $\langle \{S_{n_0}, S_{n_1}\}, \{id_n, x_n\}, \{know(Nat)\}, \rightarrow_n, S_{n_0}, \{id_n := n, x_n := n\} \rangle$, where $\rightarrow_n$ is shown in Fig. 3. The $-$ notation in $\langle -, - \rangle$ and $\langle x_n \leq id_n, - \rangle$ on transition labels denotes an empty condition or assignment.

When a process P is defined, it is deployed at a node with address ℓ, where the underlying data-link layer has capacity K and response time T_{Mac}. This is denoted by $P : (T_{Mac}, K) : \ell$. Nodes are composed using a parallel composition operator $\prod$ to form a MANET.

Definition 2: Let $P_i := \langle S_i, X_i, M_i, \rightarrow_i, s_{0_i}, \zeta_{0_i} \rangle$ denote the processes deployed at nodes $i = 1, \cdots, n$, with data-

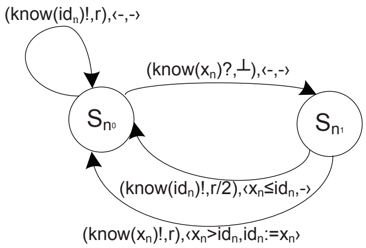

Figure 3. Extended action Markov chain of the Max protocol.

link layer (T_{Mac}, K) and network address ℓ_i, such that the X_i (and ℓ_i) are pairwise disjoint. Then $\prod_{1\leq i\leq n} P_i : (T_{Mac}, K) : \ell_i$ is a MANET.

Example. The specification of a MANET with n nodes, each deploying the Max protocol, is $\prod_{1\leq i\leq n} Max_i : (T_{Mac}, K) : A_i$ where $Max_i = \langle \{S_{i_0}, S_{i_1}\}, \{id_i, x_i\}, \{know(Nat)\}, \rightarrow_i, S_{i_0}, \{id := i, x := i\}\rangle$.

We use *constrained multi-transition labeled transition systems* to give a formal semantics of MANETs. In each process state of a network node, there may be a number of delayable send actions competing to access the data-link layer. Following [7], [9], we adopt the *race policy* mechanism for choosing the active action to execute: the action sampling the least duration wins the competition to access the data-link layer. The passive actions are executed non-deterministically. In each MANET configuration (state), there may be a number of local broadcasts (competing to access the shared communication medium). We consider an interleaving semantics between two local broadcasts which is correct from the modeling point of view, since we abstract from collisions and only consider successful receives, and also from the performance point of view, due to the memoryless property of exponential distributions. Thus when two local broadcasts $(m_1!\{\ell_1\}, r_1)$ and $(m_2!\{\ell_2\}, r_2)$ can be done in a configuration, they can be done in an arbitrary order; if we assume that $m_1!\{\ell_1\}$ is completed before $m_2!\{\ell_2\}$, then the residual time to the completion of $m_2!\{\ell_2\}$ is still exponentially distributed with rate r_2.

Since local broadcast is non-blocking, when a send action is performed by a process of a node, it is received by its data-link layer. Thus in Definition 3, we define the behavior of a process deployed at a node in a MANET regarding its data-link layer queue; when a message is broadcast, it is inserted into the queue of the data-link layer of its node, and if the queue is full, the message is dropped by the data-link layer. The data-link layer of each node transmits messages by the average time T_{Mac}. Hence the sojourn time corresponding to the transmission of a message from the data-link layer to its possible receivers (denoted by r) is an exponentially distributed random variable with rate $\frac{1}{T_{Mac}}$, called r_{Mac} hereafter. The data-link layer transmission can be synchronized with receive actions of other processes, when a sender is connected to receivers represented by a network restriction $\mathcal{C}$. The rate of a synchronization is $r_{Mac} \times P_r(\mathcal{C}) \times \beta$, where $P_r(\mathcal{C})$ is the probability of synchronization induced by $\mathcal{C}$ regarding the physical layer and mobility of nodes, and β is a normalization factor required when there is a non-deterministic choice between receive actions at a node; a choice between multiple passive actions is treated as equi-probable. The formal semantics of a MANET is given below.

Let Q_i denote the queue of the data-link layer of a node, containing i messages waiting to be transmitted. $Q_i.m(\widehat{w})$, $i < K$ denotes that a new message $m(\widehat{w})$ was received by the data-link layer and inserted at the end of its queue.

Definition 3: The semantics of a MANET $\prod_{1\leq i\leq n} P_i : (T_{Mac}, K) : \ell_i$, where $P_i = \langle S_i, X_i, M_i, \rightarrow_i, s_{0_i}, \zeta_{0_i}\rangle$, denoted as $[\![\prod_{1\leq i\leq n} P_i : (T_{Mac}, K) : \ell_i]\!]$, is a CLMS $\langle \mathbf{C}, \Sigma, \rightarrow, \mathbf{I}\rangle$, where:

- $\mathbf{C} = \overline{S \times Q} \times \Theta$ is a set of configurations, where $\overline{S \times Q} = (S_1 \times I\!\!P(M_1^K)) \times \cdots \times (S_n \times I\!\!P(M_n^K))$ represents composition of states of processes and their corresponding data-link queue, and Θ is the set of all possible environments $X \rightarrow D$ with $X = X_1 \cup \cdots \cup X_n$.
- $\Sigma = \{m(\widehat{u})!\{\ell_i\}, m(\widehat{u})! | m \in M_i \wedge 1 \leq i \leq n\}$ is a set of transition labels, ranged over by η. The label $m(\widehat{u})!\{\ell_i\}$ denotes transmission of a message $m(\widehat{u})$ from the data-link layer of a node with address ℓ_i, while $m(\widehat{u})!$ denotes transmission of message from a process to its data-link layer.
- $\rightarrow$ is a multi-set of transitions, where each transition is a tuple of $((\overline{s \times Q}, \theta), (\eta, r), \mathcal{C}, (\overline{s' \times Q'}, \theta'))$, denoted by $(\overline{s \times Q}, \theta) \xrightarrow{(\eta, r)}_{\mathcal{C}} (\overline{s' \times Q'}, \theta')$. Here $\overline{s \times Q} = ((s_1, Q_{L_1}), \cdots, (s_n \times Q_{L_n}))$, $\overline{s' \times Q'} = ((s'_1, Q'_{L'_1}), \cdots, (s'_n, Q'_{L'_n}))$ with $L_1, \ldots, L_n, L'_1, \ldots, L'_n \leq K$. Let ζ be a set of assignments, then θ' satisfies the following properties:
$$\{\forall x \in X.(\exists w \cdot x := w \in \zeta \Rightarrow (x, w\{\theta\}) \in \theta') \wedge \\ (\nexists w \cdot x := w \in \zeta \Rightarrow (x, \theta(x)) \in \theta')\}.$$
The values of variables $x \in X$ in θ are updated if there is an assignment $x := w$ in ζ, otherwise its value is unchanged. The values of $\overline{s' \times Q'}$, r, ζ and $\mathcal{C}$ are defined according to η:
  - If $\eta = m(\widehat{u})!$, then $\exists 1 \leq i \leq n : (s_i, (m(\widehat{w})!, r_i), s'_i, \langle g_i, \zeta_i\rangle) \in \rightarrow_i$ such that $\widehat{u} = \widehat{w}\{\theta\}$, $I\!\!D \models g_i\{\theta\} = true$. Consequently $\forall 1 \leq j \leq n \cdot (i \neq j) \Rightarrow (s'_j, Q'_{L'_j}) = (s_j, Q_{L_j})$ and $Q'_{L'_i} = Q_{L_i}.m(\widehat{u})$ with $L'_i = L_i + 1$, $r = r_i$, $\zeta = \zeta_i$ and $\mathcal{C} = \{\}$.
  - If $\eta = m(\widehat{u})!\{\ell_i\}$, then $\exists N = \{k | (s_k, (m(\widehat{x}_k)?, \perp), s^*_k, \langle g_k, \zeta_k\rangle) \in \rightarrow_k, I\!\!D \models g_k\{\theta\} = true, k \leq n, k \neq i\}$, and $\exists N_r, N_m$ where $N = N_r \cup N_m$, $N_r \cap N_m = \emptyset$:

* $\forall j \leq n \cdot j \notin N_r \Rightarrow s'_j = s_j \wedge j \in N_r \Rightarrow s'_j = s^*_j$, and $\forall j \leq n \cdot j \neq i \Rightarrow Q'_{L'_j} = Q_{L_j}$ and $Q'_{L'_i} = Q_{L_i - 1}$, where $Q_{L_i} = Q_{L_i - 1}.m(\widehat{u})$;
* $\zeta = \bigcup_{k \in N_r} \zeta_k\{\widehat{u}/\widehat{x}_k\} \cup \{\widehat{x}_k := \widehat{u}\}$;
* $\mathcal{C} = \bigcup_{k \in N_r} \ell_i \rightsquigarrow \ell_k$; and
* $r = r_{Mac} \times P_N(\mathcal{C}) \times \beta_{N_r}$, where β_{N_r} is the normalization factor:

$$(\prod_{\forall k \in N_r} \parallel \{(s_k, (m(\widehat{x}_k)?, \bot), s''_k, \langle g_k, \zeta\rangle) \in \rightarrow_k \mid I\!\!D \models g_k\{\theta\} = true\} \parallel)^{-1}.$$

- $\mathbf{I} = (((s_{0_1}, Q_0), \cdots, (s_{0_n}, Q_0)), \theta_0)$, with Q_0 the empty queue and $\forall x := u \in \bigcup_{1 \leq i \leq} \zeta_{0_i} \cdot (x, u) \in \theta_0$.

When $\eta = m(\widehat{u})!$, there is a process P_i which transmits message $m(\widehat{u})$ to its data-link layer. Hence the states and queues of all nodes except P_i are unchanged, while $Q'_{L'_i} = Q_{L_i}.m(\widehat{u})$ with $L'_i = L_i + 1$. When $\eta = m(\widehat{u})!\{\ell_i\}$, it means that the data-link layer of a node with address ℓ_i transmits data from its queue. For this message $m(\widehat{u})!\{\ell_i\}$, the possible set of receivers are nodes performing $m(\widehat{x})?$, computed in the set N. Among these nodes, N_r is the set of nodes that are connected and synchronized to the sender, while the nodes in N_m are disconnected or lost the data. Hence the queues of all data-link layers except node ℓ_i are unchanged, and states of nodes that actively received the data, i.e. nodes in N_r, are changed according to the receive transition in their corresponding process. The probability $P_N(\mathcal{C})$ computes the probability of this synchronization regarding the physical layer and mobility of nodes as explained in Section II-B. The normalization factor β_{N_r} is computed regarding the number of alternative receive actions in each node in N_r. The operator $\parallel S \parallel$ returns the size of the set S.

Example. Consider the MANET $\prod_{1 \leq i \leq 2} Max_i : (T_{Mac}, 1) : A_i$. The transitions of the configuration $(((S_{1_0}, \emptyset), (S_{2_1}, 1)), \{(id_1 = 1), (x_1 = 1), (id_2 = 2), (x_2 = 1)\})$ are shown in Fig. 4. The values in the queues of the data-link layers are presented abstractly by 1,2 instead of $know(1)$, $know(2)$. The values of r_{Mac10} and r_{Mac1} are $r_{Mac} \times P_{\{A_1\}}(\{\})$ and $r_{Mac} \times P_{\{A_1\}}(\{A_2 \rightsquigarrow A_1\})$ respectively. In this configuration, the node A_1 is in state 0 of its process, so it can send a message to its data-link layer since its queue is empty. Node A_2 is in state 1 of its process and its data-link layer has a message to transmit. Thus this node can send a message to its data-link layer, but since its queue is full ($K = 1$), the message is dropped. When its data-link layer transmits data, node A_1 can synchronize with it (since it is in state 0 of its process) with probability $P_{\{A_1\}}(\{A_2 \rightsquigarrow A_1\})$.

IV. Markovian Semantics of MANETs

The semantics of a MANET derived in Definition 3 is the stochastic model of the MANET: each configuration of the semantics is a state of the stochastic process, and the transitions between states are defined by transitions in the multi-set relation of the semantics. Since $r_{Mac} \in I\!\!R^{\geq 0}$, all action durations are exponentially distributed, and the total transition rate between two states is the sum of the action rates labeling transitions connecting the corresponding configurations in the semantics, as shown in the following theorem. Its proof is straightforward (cf. [9]).

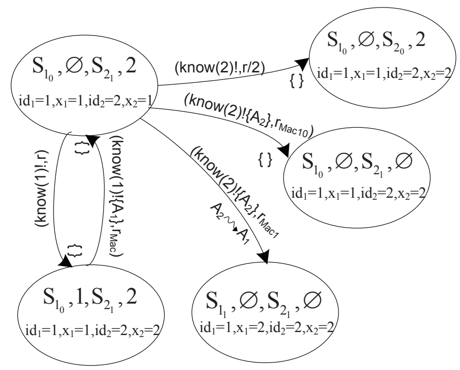

Figure 4. The transitions of $((S_{1_0}, \emptyset), (S_{2_1}, 1), \{(id_1 = 1), (x_1 = 1), (id_2 = 2), (x_2 = 1)\})$ in MANET $\prod_{1 \leq i \leq 2} Max_i : (T_{Mac}, 1) : A_i$.

Theorem 1: Given a finite MANET model $\langle \mathbf{C}, \Sigma, \rightarrow, \mathbf{I}\rangle$. Let the stochastic process $X(t)$ be defined such that $X(t) = c$ for some $c \in \mathbf{C}$ indicates that the MANET is in configuration c at time t. Then $X(t)$ is a Markov process.

If transition rates are independent of the time at which a transition occurs, the Markov process is time-homogeneous. One can use the derived stochastic model, a time-homogeneous continuous Markov chain, to evaluate the performance of a MANET in terms of different data-link layer service quality, mobility models, and protocol behavior.

Example. The MANET $\prod_{1 \leq i \leq 2} Max_i : (T_{Mac}, 1) : A_i$'s stochastic model is the same as its semantic model (the stochastic model corresponding to Fig. 4 is shown in Fig. 5). We evaluate the probability that both nodes are informed about the maximum identifier by time t. We assume that $r = 25$, that nodes run the IEEE 802.11 MAC protocol ($T_{Mac} = 1.7 \times 10^{-3}$) [1] with $P_{rcv} = 0.8$, and that nodes move according to the random waypoint model with parameters given in Section II-B (consequently $P_{UP} = 0.835$ for each pair of nodes). Given initial values, the transition rates of the stochastic model for the MANET are defined.

Let Z be a stochastic variable denoting the time at which both nodes are informed about the maximum identifier. We used the probabilistic model checker PRISM [10], to evaluate the property $P(Z < t)$ using Continuous Stochastic Logic on the derived stochastic model. The result is shown in Fig. 6. The probability that both nodes know about the maximum identifier by $t = 0.2$ when moving according to the random waypoint model is 0.8, and by $t = 0.6$ all nodes

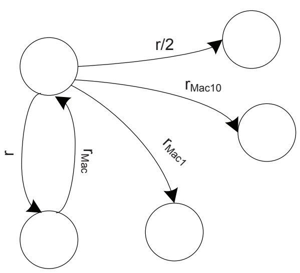

Figure 5. The stochastic model corresponding to Fig. 4.

know the maximum identifier with probability (almost) 1.

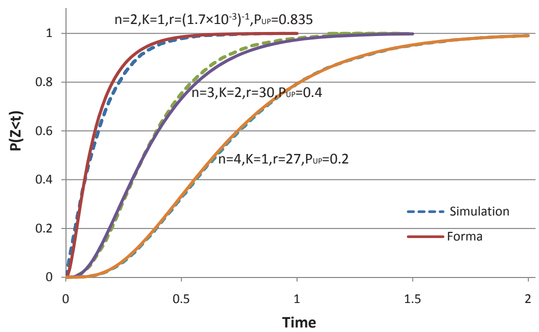

Figure 6. $P(Z < t)$ in the MANET $\prod_{1\leq i\leq n} Max_i : (T_{Mac,K}) : A_i$.

We have simulated $\prod_{1\leq i\leq 2} Max_i : (T_{Mac}, 1) : A_i$ using Java, and the result of simulation for $P(Z < t)$ is compared with the result of PRISM in Fig. 6. To simulate the Max protocol, we implemented the data-link layer as a queue with capacity K and service time[1] of $T_s = 1.7 \times 10^{-3}$. The PRISM and simulation codes can be found in [11]. We have repeated the case study for $n = 3$ when $K = 2$, $r_{Mac} = 30$, $P_{UP} = 0.4$ and $n = 4$ when $K = 1$, $r_{Mac} = 27$, $P_{UP} = 0.2$. The results of formal and simulation are shown in Fig. 6 by dashed and solid lines respectively.

V. Related Work

Different process algebras have been tailored to the specification and verification of qualitative properties of MANETs [12], [13], [14], [15], [3]. For a survey and comparison of these process algebras, we refer to [3]. There are other works apply SPIN [16], [17] and UPPAAL [18], [19] to verify qualitative and timing properties of MANETs.

In [20], the backoff algorithm in MANETs is verified using PEPA, a stochastic process algebra. In this approach the topology of the network is static and broadcast is implemented by unicasting. PRISM has been used in the verification of MANET protocols such as Bluetooth device discovery [21], the MAC protocol of IEEE 802.11 [22], IEEE 1394 root contention protocol [23], and the Zero-Conf dynamic configuration protocol for IPv4 link-local addresses [24]. These protocols (except ZeroConf) mainly belong to the data-link layer, so the effects of the data-link layer and dynamism of topology are not considered.

The only framework tailored for performance evaluation of wireless protocols (which can be used for MANETs) is [2]. In this framework, a spatial model of nodes in graphical notation is provided, in which the communication properties of nodes considering interference of adjacent nodes, distance and noise are all considered and converted into a probability for a "receive" action. A discrete Markov chain is derived for a node. A node has two modes, sending or receiving, such that the transfer to the receiving mode is performed with probability P_{rcv}. Since this approach only considers the physical characteristics of nodes and their underlying topology (which is static) and does not consider the effect of data-link layer, this model is more suitable for the verification of protocols beneath the data-link layer.

VI. Conclusions and Future Work

In this paper we provide a formal framework, based on so-called constrained labeled multi-transition systems, to evaluate MANET protocols above the data-link layer. To this aim, we consider parameters T_{Mac}, K, P_{rcv} and a mobility model to study the effects of physical characteristics of nodes, the data-link layer, and mobility of nodes. The average response time T_{Mac} of the MAC sub-layer depends on e.g. the number of nodes, queue capacity, rate of request from the upper layer, and probabilities for types of request (broadcast or unicast). One can compute this time by adjusting suitable values for these parameters. For example, in [1] this time was computed for IEEE 802.11. P_{rcv} can be approximated by a formula given in [2] (which can also be computed by simulation while computing $T_{\ell_i\ell_j,UP}$). If we assume $T_{\ell_i\ell_j,UP}$ and $T_{\ell_i\ell_j,DOWN}$ are the same for each pair of nodes, denoted by T_{UP} and T_{DOWN} respectively, then the mobility model is characterized by two values T_{UP} and T_{DOWN} which can be measured by simulation.

In defining the semantics of a MANET, we consider the behavior of a protocol regarding its data-link layer; the data-link layer has a queue of bounded capacity K and transmits messages with a rate $1/T_{Mac}$. The transmission of the data-link layer of a node is synchronized by a set of receivers such that the synchronization rate is equal to "the transmission rate"×"the probability of synchronization". We compute the probability of a synchronization regarding the physical layer and mobility of nodes; a node successfully receives with probability P_{rcv} if it is in the range of the sender. We have defined the MANET semantics by CLMSs in which transitions are labeled by an action η with a rate r and subscripted by a network restriction. The η represents transmission of data from a protocol to its data-link layer or data from a data-link layer to its potential

[1] $T_{Mac} = T_s + T_d$ where T_s and T_d are the service and queue waiting time respectively: since in this example the arrival rate of data $r = 25$ is much less than $1/T_s$, we have $T_{Mac} \approx T_s$.

receivers. Finally we derive a stochastic model of a MANET, a time-homogeneous continuous Markov chain (CTMC), under the assumption that transition rates are independent of the time at which the transitions occur. We can then exploit a probabilistic model checking tool like PRISM to evaluate MANET protocols. We can evaluate responsiveness of protocols using our framework. Besides we can extend our semantics with assigning rewards to actions (such as receive and send) to evaluate e.g. battery usage of nodes.

The definition of the semantics of MANETs based on CLMSs allows to enhance our approach to define the semantics of MANETs compositionally in the same way as [4]. We will extend our Restricted Broadcast Process Theory [3] with ideas proposed in this paper to evaluate MANETs. Then we are going to define a congruence relation on CLMSs which coincides with lumpability, an elementary notion for the aggregation of Markov chains [25].

We will build a tool to derive the Markov chain of a MANET from its specification, to apply our framework for the performance evaluation of real-world protocols.

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