On the Axiomatizability of Ready Traces, Ready Simulation and Failure Traces

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Abstract. We provide an answer to an open question, posed by van Glabbeek [4], regarding the axiomatizability of ready trace semantics. We prove that if the alphabet of actions is finite, then there exists a (sound and complete) finite equational axiomatization for the process algebra BCCSP modulo ready trace semantics. We prove that if the alphabet is infinite, then such an axiomatization does not exist. Furthermore, we present finite equational axiomatizations for BCCSP modulo ready simulation and failure trace semantics, for arbitrary sets of actions.

1 Introduction

Labeled transition systems constitute a fundamental model of concurrent computation, which is widely used in light of its flexibility and applicability. They model processes by explicitly describing their states and their transitions from state to state, together with the actions that produced them. Several notions of behavioral equivalence have been proposed, with the aim to identify those states of labeled transition systems that afford the same observations. The lack of consensus on what constitutes an appropriate notion of observable behavior for reactive systems has led to a large number of proposals for behavioral equivalences for concurrent processes.

Van Glabbeek [4] presented the linear time - branching time spectrum of 15 behavioral equivalences for finitely branching, concrete, sequential processes. For 12 equivalences in this spectrum, van Glabbeek gave an axiomatization that is sound and complete for the process algebra BCCSP modulo such an equivalence. BCCSP is built from the nil $\mathbf{0}$, alternative composition $\_ + \_$, and prefixing $a\_$, where a ranges over a nonempty set Act of actions.

For three equivalences, based on ready simulation [3, 7], failure traces [10] and ready traces [2, 11], the axiomatization in [4] includes a conditional equation. For example, for failure trace and ready trace equivalence, the axiomatizations include the conditional equation

$$I(x) = I(y) \;\Rightarrow\; a(x + y) \approx ax + ay \qquad (1)$$

where $I(p)$ is the set of possible initial actions of process p. In [4, p. 78] it is remarked that for finite alphabets, ready simulation and failure trace equivalence do allow a finite equational axiomatization.

> "As observed by Stefan Blom, if Act is finite, ready simulation equivalence can be finitely axiomatized without using conditional equations or auxiliary operators. [...] If Act is finite also failure trace equivalence has a finite equational axiomatization. *However, it is unknown whether the same holds for ready trace equivalence.*"

We present formal proofs of the observations regarding ready simulation and failure trace equivalence, for arbitrary sets of actions. The main part of this paper is devoted to answering the open question regarding ready trace equivalence.

Groote [5] introduced an infinite family of (unconditional) equations that, in the case of finitely branching processes, captures the conditional equation (1):

$$a(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z) \ \approx\ a(\sum_{i=1}^{n} b_i x_i + z) + a(\sum_{i=1}^{n} b_i y_i + z) \qquad (2)$$

for $n \in \mathbb{Z}_{>0}$. We prove that if Act consists of k elements, then actually only equation (2) for the case $n = k$ is needed, together with the equations for ready trace equivalence from [4] (excluding (1)), to obtain a (sound and complete) finite equational axiomatization for BCCSP modulo ready trace equivalence. This provides an affirmative answer to van Glabbeek's question in the case of a finite alphabet.

Van Glabbeek considers occurrences of actions in axioms as concrete action names, so that in the case of an infinite alphabet Act, an axiom such as (1) actually represents an infinite number of conditional equations, one for each $a \in Act$. In this paper we take such an occurrence of a in an axiom to represent a variable of type Act, so that (1) represents a single conditional equation. With the latter interpretation of occurrences of actions in axioms, the equational axiomatizations for 11 of the equivalences in the linear time - branching time spectrum remain finite in the case of an infinite alphabet. However, the finite equational axiomatization for ready trace equivalence given in this paper works only in the case of a finite alphabet, due to the fact that for an infinite alphabet it no longer suffices to select only one equation from the family of equations (2). We prove that in the case of an infinite alphabet, BCCSP modulo ready trace equivalence does not allow a finite equational axiomatization.

Related work: For BCCSP modulo 2-nested simulation [6], which is part of the linear time-branching time spectrum, there does not exist a finite equational axiomatization [1]; not even in the case of a finite alphabet.

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2 Preliminaries

Syntax of BCCSP BCCSP(Act) is a basic process algebra to express finite process behavior. Its syntax consists of (process) terms that are constructed from a constant $\mathbf{0}$, a binary operator $\_+\_$ called *alternative composition*, unary *prefixing* operators $c\_$, where c ranges over some nonempty set Act of *actions*, and countably infinite disjoint sets of variables $TVar$ of type term (with typical elements x, y, z) and $AVar$ of type action (with typical elements a, b). We shall use t, u, v to range over process terms and c, d, e, f to range over Act. A term is *closed* if it does not contain any variables. Closed terms will be denoted by p, q, r. A (closed) substitution maps variables in $TVar$ to (closed) BCCSP(Act) terms and variables in $AVar$ to $Act \cup AVar$ (resp. Act). For every term t and substitution σ, the term obtained by replacing every occurrence of a variable x or a in t with $\sigma(x)$ or $\sigma(a)$, respectively, will be written $\sigma(t)$.

Transition rules Intuitively, closed terms represent finite process behaviors, where $\mathbf{0}$ does not exhibit any behavior, $p + q$ is the nondeterministic choice between the behaviors of p and q, and cp can execute action c to transform into p. This intuition for the operators of BCCSP(Act) is captured, in the style of Plotkin [9], by the transition rules below, which give rise to Act-labeled transitions between closed terms.

$$\frac{}{ax \xrightarrow{a} x} \qquad \frac{x \xrightarrow{a} x'}{x + y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x + y \xrightarrow{a} y'}$$

For a closed term p, $I(p)$ denotes the set of actions c for which there exists a transition $p \xrightarrow{c} p'$ for some process term p'.

Axiomatization An *(equational) axiomatization* E over BCCSP(Act) is a collection of equations $t \approx u$. We write $E \vdash t \approx u$ if this equation can be derived from the axioms in E using the standard rules of equational logic. An axiomatization E is *sound* modulo an equivalence $\sim$ over closed terms if $E \vdash p \approx q \Rightarrow p \sim q$, and it is *complete* modulo $\sim$ if $p \sim q \Rightarrow E \vdash p \approx q$, for all closed terms p and q. An axiomatization E is ω*-complete* if for any equation $t \approx u$ such that $E \vdash \sigma(t) \approx \sigma(u)$ for all closed substitutions σ, we have $E \vdash t \approx u$.

The core equations for BCCSP(Act) are axioms A1-4 below, which are sound and complete modulo bisimulation equivalence [8].

A1	$x + y \approx y + x$
A2	$(x + y) + z \approx x + (y + z)$
A3	$x + x \approx x$
A4	$x + \mathbf{0} \approx x$

BA denotes the set of equations {A1, A2, A3, A4}. In the remainder of this paper, process terms are considered modulo associativity and commutativity of +, and modulo absorption of $\mathbf{0}$ summands (i.e., modulo A1,2,4). We use *summation* $\sum_{i=1}^{n} t_i$, with $n \in \mathbb{N}$, to denote $t_1 + \cdots + t_n$, where the empty sum denotes $\mathbf{0}$. As binding convention, alternative composition binds weaker than summation, which in turn binds weaker than prefixing.

Ready trace semantics A sequence $X_0c_1X_1 \cdots c_nX_n$ (with $n \in \mathbb{N}$), where $X_i \subseteq Act$ and $c_i \in Act$, is a *ready trace* of p_0 if $p_0 \xrightarrow{c_1} p_1 \cdots \xrightarrow{c_n} p_n$ and $I(p_i) = X_i$ for $i = 0, \ldots, n$. Two closed terms p and q are *ready trace equivalent*, denoted by $p \sim_{\mathrm{RT}} q$, if they have exactly the same ready traces. Baeten, Bergstra and Klop [2] proved that BA together with one conditional equation C1,

$$\text{C1} \qquad I(x) = I(y) \;\Rightarrow\; a(x+y) \approx ax + ay$$

is sound and complete for BCCSP(Act) modulo ready trace equivalence; see also [4]. C1 gives rise to an equality $a(p+q) \approx ap + aq$ if its condition $I(p) = I(q)$ is satisfied.

Theorem 1. *$BA \cup \{\mathrm{C1}\}$ is sound and complete for* BCCSP(Act) *modulo ready trace equivalence.*

Failure trace semantics A sequence $X_0c_1X_1 \cdots c_nX_n$ (with $n \in \mathbb{N}$), where $X_i \subseteq Act$ and $c_i \in Act$, is a *failure trace* of p_0 if $p_0 \xrightarrow{c_1} p_1 \cdots \xrightarrow{c_n} p_n$ and $I(p_i) \cap X_i = \emptyset$ for $0 = 1, \ldots, n$. Two closed terms p and q are *failure trace equivalent*, denoted by $p \sim_{\mathrm{FT}} q$, if they have exactly the same failure traces. BA and C1 together with one equation,

$$\text{FT} \qquad ax + ay \approx ax + ay + a(x+y)$$

is sound and complete for BCCSP(Act) modulo failure trace equivalence; see [4].

Theorem 2. *$BA \cup \{\mathrm{FT}, \mathrm{C1}\}$ is sound and complete for* BCCSP(Act) *modulo failure trace equivalence.*

Ready simulation A binary relation R on closed terms is a *simulation* if whenever pRq and $p \xrightarrow{a} p'$, then there is a transition $q \xrightarrow{a} q'$ such that $p'Rq'$. A simulation R is a *ready simulation* if pRq implies $I(p) = I(q)$. Two closed terms p and q are *ready simulation equivalent*, denoted by $p \sim_{\mathrm{RS}} q$, if pR_1q and qR_2p for ready simulations R_1 and R_2. BA together with one conditional equation C2,

$$\text{C2} \qquad I(y) \subseteq I(x) \;\Rightarrow\; a(x+y) \approx a(x+y) + ax$$

is sound and complete for BCCSP(Act) modulo ready simulation equivalence; see [4].

Theorem 3. *$BA \cup \{\mathrm{C2}\}$ is sound and complete for* BCCSP(Act) *modulo ready simulation equivalence.*

We take occurrences of actions in axioms (such as the a in C1 and C2) to represent variables in $AVar$.

3 Ready Traces

Groote [5] noted that C1 can be replaced by an infinite family of (unconditional) equations RT_n for $n \in \mathbb{Z}_{>0}$:

$$\mathrm{RT}_n \qquad a(\sum_{i=1}^{n}(b_ix_i + b_iy_i) + z) \;\approx\; a(\sum_{i=1}^{n} b_ix_i + z) + a(\sum_{i=1}^{n} b_iy_i + z)$$

3.1 Finite Alphabets

We prove that for a finite alphabet Act, consisting of n actions, $BA \cup \{\mathrm{RT}_n\}$ is complete for BCCSP(Act) modulo ready trace equivalence.

Lemma 1. $\{\mathrm{RT}_n, \mathrm{A3}\} \vdash \mathrm{RT}_m$ *for* $m, n \in \mathbb{Z}_{>0}$ *with* $m \leq n$.

Proof. (Sketch) Substitute b_m for b_i, x_m for x_i and y_m for y_i in RT_n, for $i = m+1, \ldots, n$. Next, apply A3 to eliminate multiple occurrences of $b_m x_m$ and $b_m y_m$ in summations. □

Proposition 1. $BA \cup \{\mathrm{RT}_n\} \vdash$

$$a(\sum_{i=1}^{n}(\sum_{j=1}^{\ell} b_i x_{ij} + \sum_{k=1}^{m} b_i y_{ik}) + z) \;\approx\; a(\sum_{i=1}^{n}\sum_{j=1}^{\ell} b_i x_{ij} + z) + a(\sum_{i=1}^{n}\sum_{k=1}^{m} b_i y_{ik} + z)$$

for $\ell, m, n \in \mathbb{Z}_{>0}$.

Proof. We take n fixed, and prove the equation by induction on $\ell + m$. The base case $\ell = m = 1$ is an instance of RT_n. We proceed with the inductive case, where $\ell + m > 2$; without loss of generality we can assume that $\ell > 1$. IH is shorthand for the induction hypothesis.

$$\begin{array}{lll}
& a(\sum_{i=1}^{n}(\sum_{j=1}^{\ell} b_i x_{ij} + \sum_{k=1}^{m} b_i y_{ik}) + z) & \\
\approx & a(\sum_{i=1}^{n}(\sum_{j=1}^{\ell-1} b_i x_{ij} + \sum_{k=1}^{m} b_i y_{ik}) + (\sum_{i=1}^{n} b_i x_{i\ell} + z)) & \\
\approx & a(\sum_{i=1}^{n}\sum_{j=1}^{\ell-1} b_i x_{ij} + \sum_{i=1}^{n} b_i x_{i\ell} + z) + a(\sum_{i=1}^{n}\sum_{k=1}^{m} b_i y_{ik} + \sum_{i=1}^{n} b_i x_{i\ell} + z) & \text{(IH)} \\
\approx & a(\sum_{i=1}^{n}\sum_{j=1}^{\ell-1} b_i x_{ij} + \sum_{i=1}^{n} b_i x_{i\ell} + z) & \\
& \quad + a(\sum_{i=1}^{n}\sum_{k=1}^{m} b_i y_{ik} + z) + a(\sum_{i=1}^{n} b_i x_{i\ell} + z) & \text{(IH)} \\
\approx & a(\sum_{i=1}^{n}\sum_{j=1}^{\ell-1} b_i x_{ij} + \sum_{i=1}^{n} b_i x_{i\ell} + z) & \\
& \quad + a(\sum_{i=1}^{n} b_i x_{i\ell} + \sum_{i=1}^{n} b_i x_{i\ell} + z) + a(\sum_{i=1}^{n}\sum_{k=1}^{m} b_i y_{ik} + z) & \text{(A3)}
\end{array}$$

$$\approx a(\sum_{i=1}^{n}\sum_{j=1}^{\ell-1} b_i x_{ij} + \sum_{i=1}^{n} b_i x_{i\ell} + \sum_{i=1}^{n} b_i x_{i\ell} + z) + a(\sum_{i=1}^{n}\sum_{k=1}^{m} b_i y_{ik} + z) \qquad \text{(IH)}$$

$$\approx a(\sum_{i=1}^{n}\sum_{j=1}^{\ell} b_i x_{ij} + z) + a(\sum_{i=1}^{n}\sum_{k=1}^{m} b_i y_{ik} + z) \qquad \text{(A3)}$$

□

Theorem 4. *Let Act consist of n actions. Then $BA \cup \{\mathrm{RT}_n\}$ is sound and complete for* BCCSP(Act) *modulo ready trace equivalence.*

Proof. Let $I(p) = I(q) = \{d_1, \ldots, d_m\}$ where $0 \leq m \leq n$. If $m = 0$, then $p \approx \mathbf{0} \approx q$ can be derived using A4. Suppose $m > 0$. Then by applying A3, p and q can be equated to closed terms of the form $\sum_{i=1}^{m}\sum_{j=1}^{\ell} d_i p_{ij}$ and $\sum_{i=1}^{m}\sum_{j=1}^{\ell} d_i q_{ij}$, respectively, for some $\ell \in \mathbb{Z}_{>0}$. Hence, by Proposition 1, for each $c \in Act$, $c(p + q) = cp + cq$ can be derived from $BA \cup \{\mathrm{RT}_m\}$. So in view of Lemma 1, each closed instantiation of C1 can be derived from $BA \cup \{\mathrm{RT}_n\}$.

By Theorem 1, $BA \cup \{\mathrm{C1}\}$ is complete for BCCSP(Act) modulo ready trace equivalence. Hence, $BA \cup \{\mathrm{RT}_n\}$ is also complete for BCCSP(Act) modulo ready trace equivalence. □

3.2 Infinite Alphabets

We prove that for an infinite alphabet Act, there does not exist a sound and complete finite equational axiomatization for BCCSP(Act) modulo ready trace equivalence. Let RT denote the set of equations $\{\mathrm{RT}_n \mid n \in \mathbb{Z}_{>0}\}$.

Corollary 1. *For any Act, $BA \cup RT$ is complete for* BCCSP(Act) *modulo ready trace equivalence.*

Proof. Let $p \sim_{\mathrm{RT}} q$. We take a nonempty, finite set $S \subseteq Act$ containing all actions that occur in p or q; clearly, p and q are BCCSP(S)-terms. Let S contain n elements. According to Theorem 4, $p \approx q$ can be derived from $BA \cup \{\mathrm{RT}_n\}$. □

The following theorem is due to Groote [5]. It does not hold for finite alphabets (cf. the example on p. 321 in [5]).

Theorem 5. *If Act is infinite, then $BA \cup RT$ is ω-complete.*

The proposition below expresses that RT_{n+1} cannot be derived from $BA \cup \{\mathrm{RT}_n\}$, for $n \in \mathbb{Z}_{>0}$. First we state without proof a simple lemma.

Lemma 2. *Let $\ell, m \in \mathbb{Z}_{>0}$ and $d_1, \ldots, d_m, e \in Act$. For closed substitutions σ,*

$$\sigma(\sum_{i=1}^{\ell}(b_i x_i + b_i y_i) + z) \sim_{\mathrm{RT}} \sum_{i=1}^{m} d_i e\mathbf{0}$$
$$\Leftrightarrow \quad \sigma(\sum_{i=1}^{\ell} b_i x_i + z) \sim_{\mathrm{RT}} \sum_{i=1}^{m} d_i e\mathbf{0} \quad \wedge \quad \sigma(\sum_{i=1}^{\ell} b_i y_i + z) \sim_{\mathrm{RT}} \sum_{i=1}^{m} d_i e\mathbf{0}$$

Proposition 2. *Let $n \in \mathbb{Z}_{>0}$. Let $d_1, \ldots, d_{n+1} \in Act$ be distinct, and let $e, f \in Act$ be distinct. Then*

$$BA \cup \{\mathrm{RT}_n\} \nvdash \ c(\sum_{i=1}^{n+1}(d_i e\mathbf{0} + d_i f\mathbf{0})) \ \approx \ c(\sum_{i=1}^{n+1} d_i e\mathbf{0}) + c(\sum_{i=1}^{n+1} d_i f\mathbf{0})$$

Proof. Let p be of the form $\sum_{j=1}^{\ell} cp_j + \sum_{k=1}^{m} cp'_k$ where $\ell, m \in \mathbb{Z}_{>0}$, $p_j \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i e\mathbf{0}$ for $j = 1, \ldots, \ell$ and $p'_k \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i f\mathbf{0}$ for $k = 1, \ldots, m$. We write $P_{n+1}(p)$ to express that p is of this particular form.

We prove that if $P_{n+1}(p)$ and $p \approx q$ can be derived by one application of an axiom in $BA \cup \{\mathrm{RT}_n\}$, then $P_{n+1}(q)$. We distinguish seven cases.

1. $p \approx q$ is derived by an application of A3.
   Then trivially $P_{n+1}(q)$.
2. $p \approx q$ is derived by an application of RT_n within a p_j or p'_k.
   Then, by the soundness of RT_n, $P_{n+1}(q)$.
3. The left-hand side $a(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z)$ of RT_n is applied to a cp_j.
   Then $\sigma(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z) = p_j \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i e\mathbf{0}$ for a closed substitution σ. By Lemma 2, $\sigma(\sum_{i=1}^{n} b_i x_i + z) \sim_{\mathrm{RT}} \sigma(\sum_{i=1}^{n} b_i y_i + z) \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i e\mathbf{0}$. This implies $P_{n+1}(q)$.
4. The left-hand side $a(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z)$ of RT_n is applied to a cp'_k.
   Then $\sigma(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z) = p'_k \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i f\mathbf{0}$ for a closed substitution σ. By Lemma 2, $\sigma(\sum_{i=1}^{n} b_i x_i + z) \sim_{\mathrm{RT}} \sigma(\sum_{i=1}^{n} b_i y_i + z) \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i f\mathbf{0}$. This implies $P_{n+1}(q)$.
5. The right-hand side $a(\sum_{i=1}^{n} b_i x_i + z) + a(\sum_{i=1}^{n} b_i y_i + z)$ of RT_n is applied to a pair of summands $cp_{j_1} + cp_{j_2}$.
   Without loss of generality we can assume that $\sigma(\sum_{i=1}^{n} b_i x_i + z) = p_{j_1}$ and $\sigma(\sum_{i=1}^{n} b_i y_i + z) = p_{j_2}$ for a closed substitution σ. Then $\sigma(\sum_{i=1}^{n} b_i x_i + z) \sim_{\mathrm{RT}} \sigma(\sum_{i=1}^{n} b_i y_i + z) \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i e\mathbf{0}$. By Lemma 2, $\sigma(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z) \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i e\mathbf{0}$. This implies $P_{n+1}(q)$.
6. The right-hand side $a(\sum_{i=1}^{n} b_i x_i + z) + a(\sum_{i=1}^{n} b_i y_i + z)$ of RT_n is applied to a pair of summands $cp'_{k_1} + cp'_{k_2}$.
   Without loss of generality we can assume that $\sigma(\sum_{i=1}^{n} b_i x_i + z) = p'_{k_1}$ and $\sigma(\sum_{i=1}^{n} b_i y_i + z) = p'_{k_2}$ for a closed substitution σ. Then $\sigma(\sum_{i=1}^{n} b_i x_i + z) \sim_{\mathrm{RT}} \sigma(\sum_{i=1}^{n} b_i y_i + z) \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i f\mathbf{0}$. By Lemma 2, $\sigma(\sum_{i=1}^{n}(b_i x_i + b_i y_i) + z) \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i f\mathbf{0}$. This implies $P_{n+1}(q)$.
7. The right-hand side $a(\sum_{i=1}^{n} b_i x_i + z) + a(\sum_{i=1}^{n} b_i y_i + z)$ of RT_n is applied to a pair of summands $cp_j + cp'_k$. This case leads to a contradiction.
   Without loss of generality we can assume that $\sigma(\sum_{i=1}^{n} b_i x_i + z) = p_j$ and $\sigma(\sum_{i=1}^{n} b_i y_i + z) = p'_k$ for a closed substitution σ. Since $p_j \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i e\mathbf{0}$, and $d_1, \ldots, d_{n+1}$ are distinct, the first identity yields $\sigma(z) \xrightarrow{d_{i_0}} r \xrightarrow{e} r'$ for some $i_0 \in \{1, \ldots, n+1\}$. Then the second identity yields $p'_1 \xrightarrow{d_{i_0}} r \xrightarrow{e} r'$. Since $e \neq f$, this contradicts the fact that $p'_1 \sim_{\mathrm{RT}} \sum_{i=1}^{n+1} d_i f\mathbf{0}$.

Concluding, if $P_{n+1}(p)$, and $p \approx q$ can be derived by an application of an axiom in $BA \cup \{\mathrm{RT}_n\}$, then $P_{n+1}(q)$. Since $\neg P_{n+1}(c(\sum_{i=1}^{n+1}(d_i e\mathbf{0} + d_i f\mathbf{0}))$ and $P_{n+1}(c(\sum_{i=1}^{n+1} d_i e\mathbf{0}) + c(\sum_{i=1}^{n+1} d_i f\mathbf{0}))$, this proves the proposition. □

Theorem 6. *If Act is infinite, then there does not exist a sound and complete finite equational axiomatization for* BCCSP(Act) *modulo ready trace equivalence.*

Proof. Let E be a finite equational axiomatization that is sound for BCCSP(Act) modulo ready trace equivalence. According to Corollary 1, $BA \cup RT$ is complete for BCCSP(Act) modulo ready trace equivalence. Hence, all closed instantiations of equations in E can be derived from $BA \cup RT$. By Theorem 5, $BA \cup RT$ is ω-complete, so the equations in E can be derived from $BA \cup RT$. Since each of these derivations requires only a finite number of applications of axioms in $BA \cup RT$, and E is finite, the equations in E can be derived from a finite subset of $BA \cup RT$. In view of Lemma 1, this means that the equations in E can be derived from $BA \cup \{\mathrm{RT}_n\}$ for some $n \in \mathbb{Z}_{>0}$. So by Proposition 2,

$$c(\sum_{i=1}^{n+1}(d_i e\mathbf{0} + d_i f\mathbf{0})) \;\approx\; c(\sum_{i=1}^{n+1} d_i e\mathbf{0}) + c(\sum_{i=1}^{n+1} d_i f\mathbf{0})$$

with $d_1, \ldots, d_{n+1}$ distinct and e, f distinct, cannot be derived from E. Hence, E is not complete for BCCSP(Act) modulo ready trace equivalence. □

4 Ready Simulation

We prove that the conditional axiom C2 can be replaced by a single unconditional equation

$$\text{RS} \qquad a(x + by + bz) \approx a(x + by + bz) + a(x + by)$$

It is not hard to see that RS is sound modulo ready simulation equivalence.

Theorem 7. $BA \cup \{\mathrm{RS}\}$ *is sound and complete for* BCCSP(Act) *modulo ready simulation equivalence.*

Proof. Let $I(q) \subseteq I(p)$, where q is of the form $\sum_{i=1}^{m} b_i q_i$. We prove that $a(p+q) \approx a(p+q) + ap$ can be derived from $BA \cup \{\mathrm{RS}\}$, by induction on m. The base case $m = 0$ is trivial, using A3,4. We focus on the inductive case, where $m > 0$.

Since $I(q) \subseteq I(p)$, p contains a summand $b_m p'$. Hence,

$$\begin{aligned} a(p+q) &\approx a(p+q) + a(p + \textstyle\sum_{i=1}^{m-1} b_i q_i) && \text{(RS)} \\ &\approx a(p+q) + a(p + \textstyle\sum_{i=1}^{m-1} b_i q_i) + ap && \text{(IH)} \\ &\approx a(p+q) + ap && \text{(RS)} \end{aligned}$$

This completes the inductive argument. Concluding, each closed instance of C2 can be derived from $BA \cup \{\mathrm{RS}\}$.

By Theorem 3, $BA \cup \{\mathrm{C2}\}$ is complete for BCCSP(Act) modulo ready simulation equivalence. Hence, $BA \cup \{\mathrm{RS}\}$ is also complete for BCCSP(Act) modulo ready simulation equivalence. □

5 Failure Traces

Theorem 8. *$BA \cup \{\mathrm{FT}, \mathrm{RS}\}$ is sound and complete for* BCCSP(Act) *modulo failure trace equivalence.*

Proof. Let $I(p) = I(q)$. Then $a(p+q) \sim_{\mathrm{RS}} a(p+q) + ap + aq$, so according to Theorem 7, $a(p+q) \approx a(p+q) + ap + aq$ can be derived from $BA \cup \{\mathrm{RS}\}$. By FT, $a(p+q) + ap + aq \approx ap + aq$. Concluding, each closed instance of C1 can be derived from $BA \cup \{\mathrm{FT}, \mathrm{RS}\}$.

By Theorem 2, $BA \cup \{\mathrm{FT}, \mathrm{C1}\}$ is complete for BCCSP(Act) modulo failure trace equivalence. Hence, $BA \cup \{\mathrm{FT}, \mathrm{RS}\}$ is also complete for BCCSP(Act) modulo failure trace equivalence. □

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